$e^{-2}) = \dfrac{\frac{2}{e^{-4}}}{-1} = -2e^4 < 0$

$e^{-4}) = \dfrac{\frac{2}{e^{-8}}\cdot(-1)}{+27} \qquad \dfrac{2}{e^{-8}}\cdot\dfrac{1}{27} > 0$

$(0, e^{-3}) \cup (e^{-1}, \infty) \Rightarrow$ KONVEKSNA

$(e^{-3}, e^{-1}) \Rightarrow$ KONKAVNA

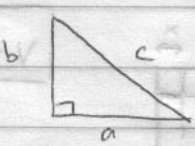
) DOLOČI PRAVOKOTNI TRIKOTNIK Z OBSEGOM 10, KI IMA NAJVEČJO PLOŠČINO!

$\boxed{pl = \frac{ab}{2}}$ PLOŠČINA $\perp$ TRIKOTNIKA

$\boxed{o = a+b+c}$ OBSEG $\perp\ \Delta$

$c^2 = a^2 + b^2$

$c = \sqrt{a^2+b^2}$



$10 = a + b + \sqrt{a^2+b^2}$

$10 - a - b = \sqrt{a^2+b^2} \quad /^2$

$100 + \cancel{a^2} + \cancel{b^2} - 20a - 20b + 2ab = \cancel{a^2} + \cancel{b^2}$

$100 - 20a = 20b - 2ab$

$100 - 20a = b(20-2a)$

$b = \dfrac{100-20a}{20-2a} \quad /:2$

$b = \dfrac{50-10a}{10-a}$

$pl = \dfrac{a\cdot\frac{50-10a}{10-a}}{2} = \dfrac{50a-10a^2}{20-2a} \rightarrow$ IŠČEMO EKSTREM FUNKCIJE pl NA INTERVALU $[0, 5]$ 3

$pl' = \dfrac{(50-20a)(20-2a)-(50a-10a^2)(-2)}{(20-2a)^2} =$

$= \dfrac{1000-400a-\cancel{100a}+40a^2+\cancel{100a}-20a^2}{(20-2a)^2} =$

$= \dfrac{20a^2-400a+1000}{(20-2a)^2}$

$$f''(e^{-2}) = \frac{\frac{2}{e^{-4}}}{-1} = -2e^4 < 0$$

$$f''(e^{-4}) = \frac{\frac{2}{e^{-8}}\cdot(-1)}{-27} \qquad \frac{2}{e^{-8}}\cdot\frac{1}{27} > 0$$

$(0, e^{-3}) \cup (e^{-1}, \infty) \Rightarrow$ KONVEKSNA

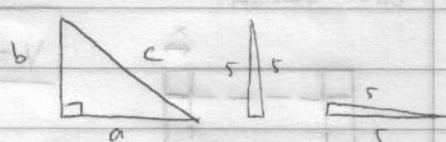
$(e^{-3}, e^{-1}) \Rightarrow$ KONKAVNA

4. DOLOČI PRAVOKOTNI TRIKOTNIK Z OBSEGOM 10, KI IMA NAJVEČJO PLOŠČINO!

$\boxed{pl = \frac{ab}{2}}$ PLOŠČINA $\perp$ TRIKOTNIKA

$\boxed{o = a+b+c}$ OBSEG $\perp$ Δ

$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2+b^2}$$



$$10 = a + b + \sqrt{a^2+b^2}$$
$$10-a-b = \sqrt{a^2+b^2} \quad /^2$$
$$100 + \cancel{a^2} + \cancel{b^2} - 20a - 20b + 2ab = \cancel{a^2} + \cancel{b^2}$$
$$100 - 20a = 20b - 2ab$$
$$100 - 20a = b(20-2a)$$
$$b = \frac{100-20a}{20-2a} \quad \Big/ \begin{matrix} :2 \\ :2 \end{matrix}$$
$$b = \frac{50-10a}{10-a}$$

$$pl = \frac{a\cdot\frac{50-10a}{10-a}}{2} = \frac{50a-10a^2}{20-2a}$$ $\rightarrow$ IŠČEMO EKSTREM FUNKCIJE pl NA INTERVALU $[0, 5]$

$$pl' = \frac{(50-20a)(20a-2a)-(50a-10a^2)(-2)}{(20-2a)^2} =$$

$$= \frac{1000-400a-\cancel{100a}+40a^2+\cancel{100a}-20a^2}{(20-2a)^2} =$$

$$= \frac{20a^2-400a+1000}{(20-2a)^2}$$

$\boxed{p\,' = 0}$ $\qquad 20a^2 - 400a + 1000 = 0$

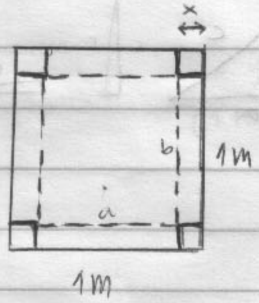
$$a^2 - 20a + 50 = 0$$

$$a_{1,2} = \frac{20 \pm \sqrt{400 - 200}}{2} = \frac{20 \pm \sqrt{200}}{2} = 10 \pm \sqrt{50}$$

$a = 10 - \sqrt{50}$ $\qquad$ a NE SME BITI VEČJI OD 10!

$a = 10 - 3\sqrt{2}$

(5) IZ KVADRATNEGA KARTONA, S STRANICO 1 m, NAREDIMO ŠKATLO BREZ ZGORNJE PLOSKVE TAKO, DA PRI OGLJIŠČIH IZREŽEMO 4 MANJŠE KVADRATKE. KAKO VELIKE KVADRATE MORAMO IZREZATI, DA JE PROSTORNINA ŠKATLE ČIM VEČJA?



$\boxed{V = a \cdot b \cdot c}$

$a + 2x = 1 \qquad\qquad b + 2x = 1$

$a = 1 - 2x \qquad\qquad b = 1 - 2x$

$$V = (1-2x)(1-2x) \cdot x$$

IŠČEMO MAKSIMUM FUNKCIJE NA INTERVALU $\left[0, \frac{1}{2}\right]$

$V = (1 - 2x - 2x + 4x^2)\,x =$

$= (1 - 4x + 4x^2)\,x =$

$= x - 4x^2 + 4x^3$

$V' = 1 - 8x + 12x^2$

$\boxed{V' = 0}$

$\Downarrow$

$12x^2 - 8x + 1 = 0$

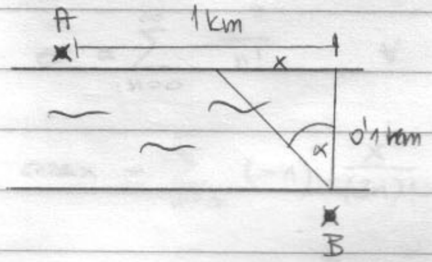
$$x_{1,2} = \frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 12}}{12 \cdot 2} = \frac{8 \pm \sqrt{16}}{24}$$

$x_1 = \frac{1}{2} \qquad \boxed{x_2 = \frac{1}{6}}$

$\downarrow$

$V = 0$

(6.) KRAJA A IN B LEŽITA OB 100 m ŠIROKI REKI (VSAK NA ~~HRIBU~~ BREGU) IN STA MED SEBOJ ODDALJENA 1 km. PO KAKŠNI POTI NAJ GREMO IZ A DO B, DA BOMO ČIMHITREJŠI, ČE 3x HITREJE HODIMO KOT PLAVAMO?



$s = v \cdot t$

$t = \frac{s}{v}$

$T = t_1 + t_2$

$T = \frac{1-x}{3v} + \frac{\sqrt{x^2+(0'1)^2}}{v}$

$T = \frac{(1-x) + 3\sqrt{x^2+(0'1^2)}}{3v}$

IŠČEMO MIN FUNKCIJE NA INTERVALU $[0, 1]$

$$T' = \frac{-1 + 3\,\frac{1}{2}\,(x^2+0'1^2)^{-\frac{1}{2}} \cdot 2x}{3v}$$

$$-1 + \frac{3}{2}(x^2+0'1^2)^{-\frac{1}{2}}\, 2x = 0$$

$$-1 + \frac{3}{2}\,\frac{1}{\sqrt{x^2+0'1^2}} \cdot 2x = 0$$

$$-\sqrt{x^2+0'1^2} + 3x = 0$$

$$x^2 + 0'1^2 = 9x^2$$

$$0'1^2 = 8x^2$$

$$0'1 = 2x\sqrt{2}$$

$$x = \frac{0'1}{2\sqrt{2}}$$

L'Hospitalovo pravilo

16.02.2005

$$f(x)\cdot g(x) = \frac{f(x)}{g(x)}$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$$

odvajamo posebej števec in posebej imenovalec

* če $f(a)=0=g(a)$
* če $f(a)=\infty=g(a)$

Npr.: $\lim_{x\to 1}\frac{x^3-2x^2-x+2}{x^3-7x+6} = \left(\frac{0}{0}\right)\lim_{x\to 1}\frac{3x^2-4x-1}{3x^2-7} = \frac{-2}{-4} = \frac{1}{2}$

$$\lim_{x\to 0}\frac{x\cos - \sin x}{x^3} = \left(\frac{0}{0}\right)\lim\frac{\cos x + x(-\sin x) - \cos x}{3x^2} =$$

$$= \lim_{x\to 0}\frac{-\sin x\, \cancel{x}}{3x^{\cancel{2}}} \; {=1} = \left(\frac{0}{0}\right)\lim_{x\to 0}\frac{-\cos x}{3} = -\frac{1}{3}$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x - \sin x}{x - \sin x} = \left(\frac{0}{0}\right)\lim\frac{\frac{1}{\cos^2 x} - \cos x}{1-\cos x} = \left(\frac{0}{0}\right)\lim$$

$$\lim_{x\to 0}\frac{\frac{1-\cos^3 x}{\cos^2 x}}{1-\cos x} = \lim_{x\to 0}\frac{1-\cos^3 x}{\cos^2 x - \cos^3 x} = \left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\frac{3\cos^2 x\cdot\sin x}{-2\cos x\,\sin x + 3\cos^2 x\sin x} = \lim_{x\to 0}\frac{3\cos^2 x\sin x}{\cos x\sin x(-2+3\cos x)} = \frac{3}{1} = 3$$

za L'Hospitalovo p. rabimo ulomek, če ga nimamo ga naredimo

$$\lim_{x\to 0}(1-\cos x)\cdot\operatorname{ctg} x = \lim_{x\to 0}\frac{(1-\cos x)\cos x}{\sin x} = \left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\frac{\sin x\cos x + (1-\cos x)(-\sin x)}{\cos x} = \lim_{x\to 0}\frac{\sin x\cos x - \sin x + \sin x\cos x}{\cos x} =$$

$$= \frac{0}{1} = 0$$

$$\lim_{x\to 0} x \ln x^2 = \lim_{x\to 0} \frac{\ln x^2}{x^{-1}} = \left(\frac{\infty}{\infty}\right) \lim \frac{\frac{1}{x^2} 2x}{-1 x^{-2}} = \ldots$$

$$= \lim_{x\to 0} \frac{\frac{1\cdot 2x}{x^2}}{-1x^{-2}} = \lim_{x\to 0} \frac{1\cdot 2x}{x^2(-1x^{-2})} = \lim_{x\to 0} \frac{2x}{-1} \to 0 \quad \boxed{-1x^{-1-1} = -1x^{-2}}$$

$$\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right) = \lim_{x\to 1}\left(\frac{x\ln x - (x-1)}{\ln x (x-1)}\right) - \left(\frac{0}{0}\right) =$$

$$\lim_{x\to 1}\left(\frac{\ln x + x\frac{1}{x} - 1}{\frac{1}{x}(x-1) + \ln x 1}\right) = \lim_{x\to 1}\left(\frac{\ln x}{1 - \frac{1}{x} + \ln x}\right) - \left(\frac{0}{0}\right) \lim_{x\to 1}\left(\frac{\frac{1}{x}}{x^{-2} + \frac{1}{x}}\right) = \frac{1}{2}$$

$$a = \lim_{x\to\infty}\left(\frac{x+1}{x+2}\right)^{5x} \Big/ \ln \quad \ln a = \lim_{x\to\infty} \ln\left(\frac{x+1}{x+2}\right)^{5x} =$$

$$= \lim_{x\to\infty} 5x \ln\left(\frac{x+1}{x+2}\right) = 5\lim_{x\to\infty} \frac{\ln\left(\frac{x+1}{x+2}\right)}{x^{-1}} = \left(\frac{0}{0}\right)^{\ln 1 = 0} = 5\lim_{x\to\infty} \frac{\frac{1}{\frac{x+1}{x+2}} \cdot \frac{1(x+2)-(x+1)\cdot 1}{(x+2)^2}}{-x^{-2}}$$

$$\to 5\lim_{x\to\infty} \frac{\frac{x+2}{x+1} \cdot \frac{1}{(x+2)^2}}{-x^{-2}}$$

$$= 5\lim_{x\to\infty} \frac{-1}{\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)} = -5 \Rightarrow a = e^{-5}$$

$0 = \frac{1}{x}$, $0 = \frac{2}{x}$

$$\lim_{x\downarrow 0} x^{\sin x} = a \quad \Big/ \ln$$

LAHKO GA DAMO SPREDI

$$\lim_{x\downarrow 0} \ln x^{\sin x} = \lim_{x\downarrow 0} \sin x \ln x = \lim_{x\downarrow 0} \frac{\ln x}{\sin x} = \left(\frac{\infty}{\infty}\right) \lim_{x\downarrow 0} \frac{\frac{1}{x}}{\sin^{-2} x \cdot \cos x} =$$

$$\lim_{x\downarrow 0} \frac{-\sin^2 x}{x\cos x} = \left(\frac{0}{0}\right) \lim_{x\downarrow 0} \frac{-2\sin x \cdot \cos x}{\cos x - x\sin x} = \frac{0}{1} = a \quad \boxed{a = e^0 = 1}$$

$$\lim_{x\to 1}\left(\operatorname{tg}\frac{\pi x}{4}\right)^{\operatorname{tg}\frac{\pi x}{2}} = a \quad \Big/ \ln$$

$$\ln a = \lim_{x\to 1} \ln\left(\operatorname{tg}\frac{\pi x}{4}\right)^{\operatorname{tg}\frac{\pi x}{2}} = \lim_{x\to 1} \overset{\infty}{\operatorname{tg}\frac{\pi x}{2}} \overset{0}{\ln \operatorname{tg}\frac{\pi x}{4}} = \lim_{x\to 1} \frac{\ln \operatorname{tg}\frac{\pi x}{4}}{\operatorname{ctg}\frac{\pi x}{2}} =$$

$$= \lim_{x\to 1} \frac{\frac{1}{\operatorname{tg}\frac{\pi x}{4} \cdot \cos^2\frac{\pi x}{4}} \cdot \frac{\pi}{4}}{\frac{-1}{\sin^2\frac{\pi x}{2}} \cdot \frac{\pi}{2}} = \lim_{x\to 1} \frac{-\sin^2\frac{\pi x}{2} \cdot 2}{\operatorname{tg}\frac{\pi x}{4} \cdot \cos^2\frac{\pi x}{4} \cdot 4 \cdot 2} = \frac{-1}{1 \cdot \frac{2}{4} \cdot 2} = -1$$

asimtota: $y = mx + k$ je asimtota funkcije $f(x)$, če je $k = \lim_{x \to \pm\infty} \frac{f(x)}{x}$

$$m = \lim_{x \to \pm\infty} (f(x) - kx)$$

lahko tud tko!

$$f(x) = \frac{x^2}{\sqrt{x^2-1}} : k = \lim_{x \to \infty} \frac{\frac{x^2}{\sqrt{x^2-1}}}{x} = \lim_{x \to \infty} \frac{x}{\sqrt{x^2-1}} \begin{matrix} : x \\ : \sqrt{x^2} \end{matrix} =$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{1 - 1/x^2}} = 1$$

$$m = \lim_{x \to \infty} \left(\frac{x^2}{\sqrt{x^2-1}} - x \right) = \lim_{x \to \infty} \frac{x^2 - x\sqrt{x^2-1}}{\sqrt{x^2-1}} =$$

$$= \lim_{x \to \infty} \frac{(x^2 - x\sqrt{x^2-1})(x^2 + x\sqrt{x^2-1})}{\sqrt{x^2-1}\,(x^2 + x\sqrt{x^2-1})} = \lim_{x \to \infty} \frac{x^4 - x^2(x^2-1)}{\sqrt{x^2-1}\,(x^2 + x\sqrt{x^2-1})} =$$

$$= \lim_{x \to \infty} \frac{x^2 \quad : x^2}{\sqrt{x^2-1}\,(x^2 + x\sqrt{x^2-1}) \quad : x^3} = 0$$

$y = x$ je asimtota funkcije $\frac{x^2}{\sqrt{x^2-1}}$

$y = -x$ — || —, ker je $f(x)$ soda funkcija

$$f(x) = e^{-x^2} + 2$$

$$k = \lim_{x \to \infty} \frac{e^{-x^2} + 2}{x} = \left(\frac{2}{\infty} \right) = 0$$

$$m = \lim_{x \to \infty} (e^{-x^2} + 2 - 0 \cdot x) = 2$$

f je soda funk. $\Rightarrow y = 2$

Za funkcijo ne veš ali je soda ali liha zato izr. obe vrednosti

$f(x) = \ln(1+e^{-x})$

$$k = \lim_{x\to\infty} \frac{\ln(1+e^{-x})}{x} = \frac{(\ln 1)}{\infty} = 0$$

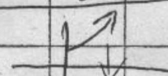
$$n = \lim_{x\to\infty} \left(\ln(1+e^{-x})\right) = \ln 1 = 0$$

1 MOŽNOST

$$k = \lim_{x\to-\infty} \frac{\ln(1+e^{-x})}{x} = \left(\frac{\infty}{\infty}\right) \lim \frac{\frac{1}{1+e^{-x}}(-e^{-x})}{1} =$$

$$= \lim_{x\to-\infty} \frac{-e^{-x} \;/:e^{x}}{1+e^{-x} \;/:e^{x}} = \lim_{x\to-\infty} \frac{-1}{e^{x}+1} = \frac{-1}{1} = -1$$

$\boxed{\text{OPA } \ln a + \ln b = \ln a \cdot b}$



$$n = \lim_{x\to-\infty} \left(\ln(1+e^{-x}) + x\right) = \lim_{x\to-\infty} \left(\ln(e^{x}+1)\right) = \ln 1 = 0$$

$L: y = -x \quad D: y = 0$

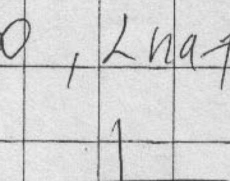
Nariši graf funk. $f(x) = \frac{\ln x}{x}$ $D_f = (0, \infty)$

NIČLA: $x = 1$

asimptota: $k = \lim_{x\to\infty} \frac{\ln x}{x^2} = \left(\frac{\infty}{\infty}\right) \lim_{x\to\infty} \frac{1/x}{2x} = \lim_{x\to\infty} \frac{1}{2x^2} = 0$

desna asimptota $y = 0$, L na funk. ni def.

$$\lim_{x\downarrow 0} \frac{\ln x}{x} = -\infty$$



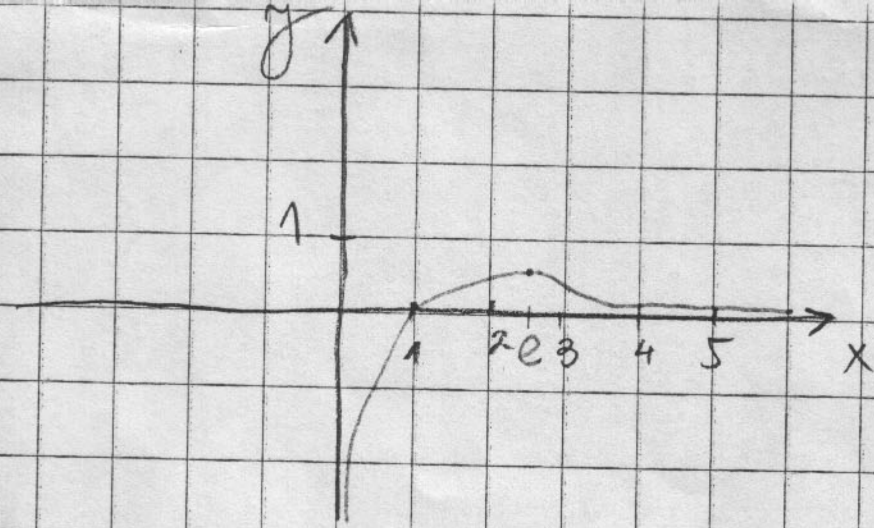
$$f'(x) = \frac{1/x \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$$

$f'(0) =$

$1 - \ln x = 0$

$\ln x = 1$

$x = e$ kob. est.



3. kolokvij 5-april od 11 VP

15 mar odpadejo predavanja imamo vaje

r = razdalja med 0 in 1

kot cos

graf v polarnih koor. Nariši graf funkcije $r = \cos 2\varphi$

Pogoj $r \geq 0$

$\cos 2\varphi \geq 0$

$\cos\alpha \geq 0 \longleftrightarrow \frac{\pi}{2} \geq \alpha \geq -\frac{\pi}{2}$

$\sin\alpha \geq 0 \longleftrightarrow \pi \geq \alpha \geq 0$

$\pi/2 + 2\pi k \geq 2\varphi \geq -\pi/2 + 2\pi k \quad k \in \mathbb{Z}$

2 pogoja, če bi blo 5, pol 5 pogojev

① POGOJ
$k = 0 \quad \frac{\pi}{2} \geq 2\varphi \geq -\frac{\pi}{2} \quad /:2$

$\frac{\pi}{4} \geq \varphi \geq -\frac{\pi}{4}$ pogoj

② POGOJ
$k = 1 \quad \frac{\pi}{2} + 2\pi \geq 2\varphi \geq -\frac{\pi}{2} + 2\pi \quad /:2$

$\frac{\pi}{4} + \pi \geq \varphi \geq -\frac{\pi}{4} + \pi$ pogoj

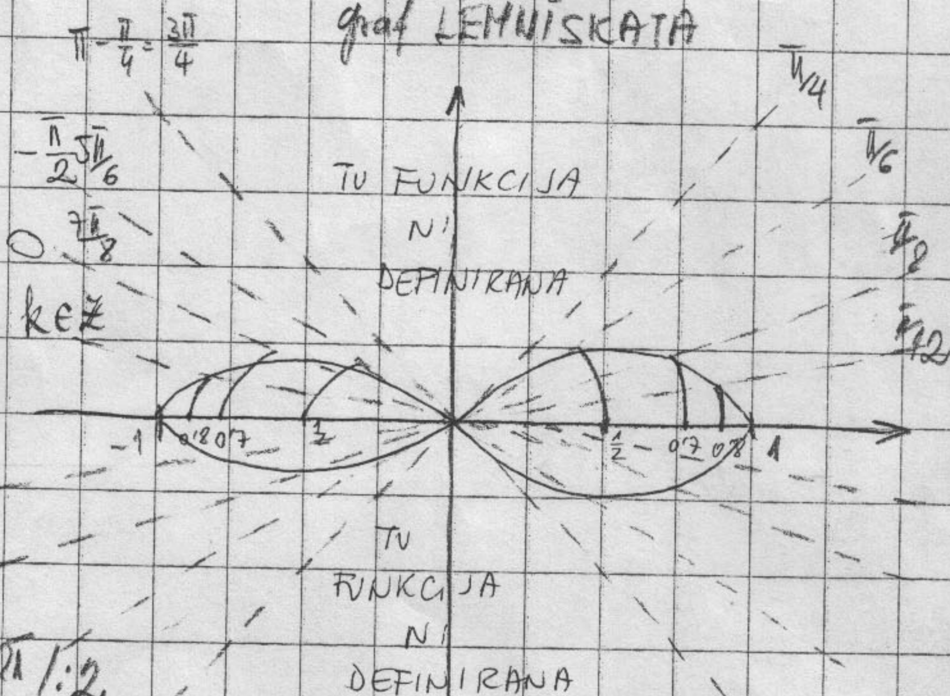
③ POGOJ
$k = 2 \quad \frac{\pi}{2} + 4\pi \geq 2\varphi \geq -\frac{\pi}{2} + 4\pi \quad /:2$

$\frac{\pi}{4} + 2\pi \geq \varphi \geq -\frac{\pi}{4} + 2\pi$

enak pogoju ①

sin in cos sta periodična na 2π!

graf LEMNISKATA



riši s šestilom (BALONČEK)

POM. RAČUNI

$r = \cos 2\cdot 0 = 1$

$r = \cos 2\cdot \pi/12 = \cos \pi/6 \doteq \frac{\sqrt{3}}{2} \doteq 0'8$

$r = \cos 2\cdot \pi/8 = \cos \pi/4 = \frac{\sqrt{2}}{2} \doteq 0'7$

$r = \cos 2\cdot \pi/6 = \cos \pi/3 = \frac{1}{2}$

$r = \cos 2\cdot \pi/4 = \cos \frac{\pi}{2} = 0$

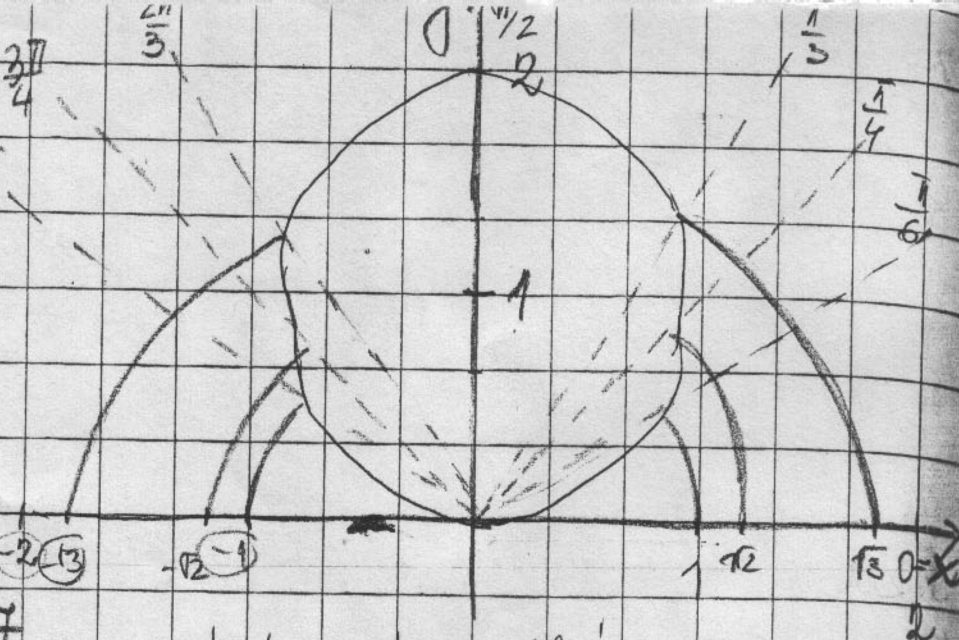
$r = \cos 2(\pi - \alpha) = \cos(2\pi - 2\alpha) = \cos 2\alpha$

$r = 2\sin\varphi$

$2\sin\varphi \geq 0 /:2$

$\sin\varphi \geq 0$

$\pi \geq \varphi \geq 0$



kot en krog pride!

$r = 2\sin 0 = 0$

$r = 2\sin \pi/6 = 2 \cdot \frac{1}{2} = 1$

$r = 2\sin \pi/4 = 2\frac{\sqrt{2}}{2} = \sqrt{2} \doteq 1'4$

$r = 2\sin \pi/3 = 2\frac{\sqrt{3}}{2} = \sqrt{3} \doteq 1'7$

$r = 2\sin \pi/2 = 2 \cdot 1 = 2$

Ali je krog?

$r^2 = 2r\sin\varphi \rightarrow x^2 + y^2 = 2y \Rightarrow x^2 + y^2 - 2y + 1 = 0 + 1$

$x = r\cos\varphi$

$y = r\sin\varphi$

$x^2 + y^2 = r^2\cos\varphi + r^2\sin\varphi = r^2$

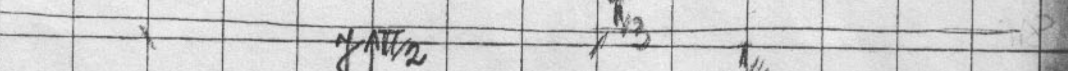
$x^2 + (y-1)^2 = 1$ enačba za krožnico

$r = 1 + \cos\varphi$

$1 + \cos\varphi \geq 0$

$\cos\varphi \geq -1$ za vse φ

upoštevamo celo defin. območje



$r = 1 + \cos 0 = 2$

$r = 1 + \cos \pi/6 = 1 + \frac{\sqrt{3}}{2} \doteq 1'8$

$r = 1 + \cos \pi/4 \doteq 1'7$

$r = 1 + \cos \pi/3 \doteq 1'5$

$r = 1 + \cos \frac{2\pi}{3} = \frac{1}{2}$

$r = 1 + \cos \frac{3\pi}{4} \doteq 0'3$

$r = 1 + \cos \frac{5\pi}{6} = 0'2$

$r = 1 + \cos \pi = 0$

SRČNICA ali

KARDIOIDA

VARKA

φ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$

$\sin 2\varphi \geq 0$

$\pi + 2k\pi \geq 2\varphi \geq 0 + 2k\pi$

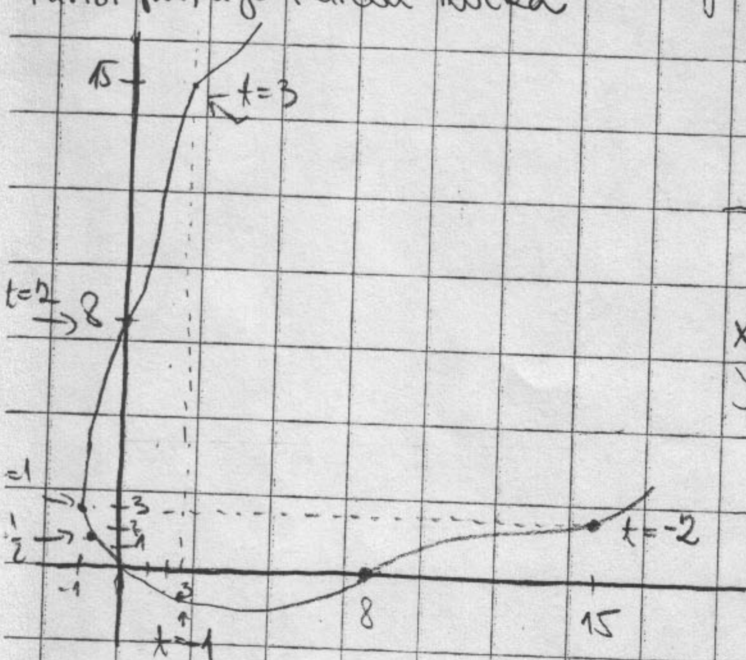
① $k = 0$ $\pi \geq 2\varphi \geq 0$, $\frac{\pi}{2} \geq \varphi \geq 0$

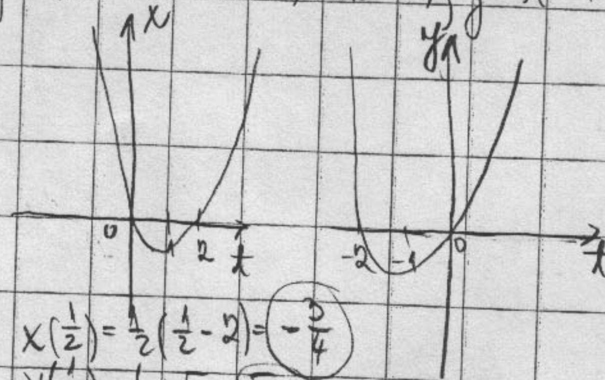
② $k = 1$ $3\pi \geq 2\varphi \geq 2\pi$

$\frac{3\pi}{2} \geq \varphi \geq \pi$

Nariši parametrično podano krivuljo $x = t^2 - 2t = t(t-2)$; $y = t^2 + 2t = t(t+2)$

nariši pot, ki jo naredi točka





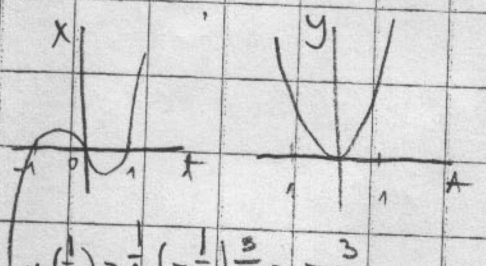
(ni lokalnih ekstremov)

$x(\frac{1}{2}) = \frac{1}{2}(\frac{1}{2} - 2) = -\frac{3}{4}$

$y(\frac{1}{2}) = \frac{1}{2} \cdot \frac{5}{2} = 5/4$

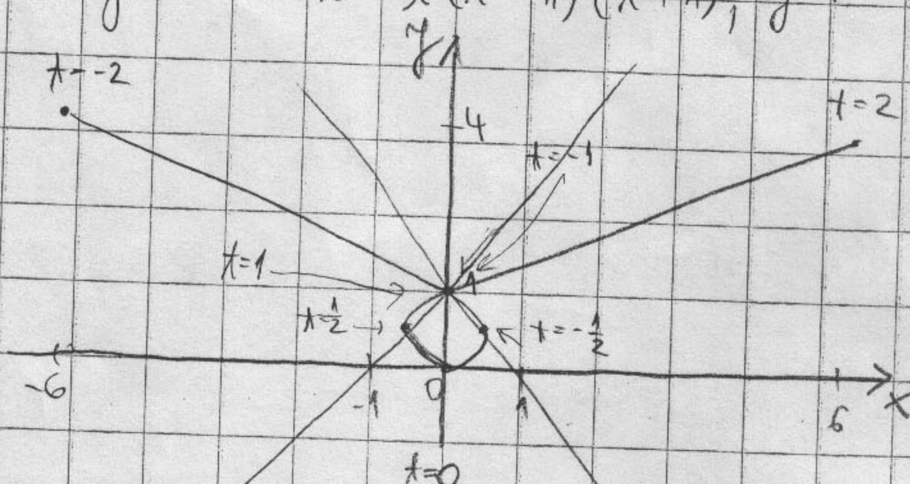
Nariši parametrično podano krivuljo $x = t^3 - t = t(t-1)(t+1)$; $y = t^2$

!2 ekstrema!



$x(\frac{1}{2}) = \frac{1}{2}(-\frac{1}{2})\frac{3}{2} = -\frac{3}{8}$

$y(\frac{1}{2}) = \frac{1}{4}$



Izračunaj tangenti na krivuljo v točkah, kjer je $y = 1$

$y = 1 \to t = \pm 1$

$k = \frac{y'(t_0)}{x'(t_0)}$ odvajaj om. enačbe $x'(t) = 3t^2 - 1$, $y'(t) = 2t$

① $t = 1$

$k = \frac{y'(1)}{x'(1)} = \frac{2}{2} = 1$

$y - 1 = 1(x - 0)$

$y = x + 1$ premica

② $t = -1$

$k = \frac{y'(-1)}{x'(-1)} = \frac{2}{-2} = -1$

$y - 1 = -1(x - 0)$

$y = -x + 1$

pravokotni premici, krivulja seka tudi samo sebe pod pravim kotom

Izračunaj tangento:

$x = t^2 - 2t$

$y = t^2 + 2t$

tangento v tč $x = 0$

$k = \frac{y'(t_0)}{x'(t_0)}$

$x' = 2t - 2$

$y' = 2t + 2$

$t^2 - 2t = 0$

$t(t-2) = 0$

$t_1 = 0$

$t_2 = 2$

① $t = 0$

$k_1 = \frac{y'(0)}{x'(0)} = \frac{2}{-2} = -1$

$x_1 = 0$

$x_2 = 0$

$y - y_1 = k_1(x - x_1)$

$y = -x$

simetrala lihih kvadrantov

② $t = 2$

$k_2 = \frac{y'(2)}{x'(2)} = \frac{6}{2} = 3$

$x_2 = 0$

$y_2 = 8$

$y - y_1 = k_2(x - x_1)$

$y - 8 = 3(x - 0)$

$y = 3x + 8$

Izračunaj tangento:

$r = 1 + \cos\varphi$

$\varphi = 0$

$\varphi = \pi/2$

$x' = -\sin\varphi(\cos\varphi) + (-\sin\varphi)(1 + \cos\varphi)$

$y' = -\sin\varphi(\sin\varphi) + (1 + \cos\varphi)\cos\varphi$

vzame r $x = r\cos\varphi = (1 + \cos\varphi)(\cos\varphi)$

$y = r\sin\varphi = (1 + \cos\varphi)(\sin\varphi)$

① $\varphi = 0$

$k_1 = \frac{y'(0)}{x'(0)} = \frac{2}{0} = \infty$

$x_1 = 2$

$y_1 = 0$

$x = 2$ tang

② $\varphi = \pi/2$

$r = 2\sin\varphi$

$\varphi = 0$

$\varphi = \pi/4$

$\varphi = \pi/2$

$$x' = 2\cos\varphi\cos\varphi + 2\sin\varphi(-\sin\varphi)$$

$$y' = 2\cos\varphi\sin\varphi + 2\sin\varphi\cos\varphi$$

(1) $k_1 = \frac{y'(0)}{x'(0)} = \frac{0}{2} = 0$

$\pi/4$ vstavi v $x = r\cos\varphi = 2\sin\varphi\cos\varphi$

$y = r\sin\varphi = 2\sin\varphi\sin\varphi$

dobiš točko 1

$y_1 = 0$ $\quad y = 0$

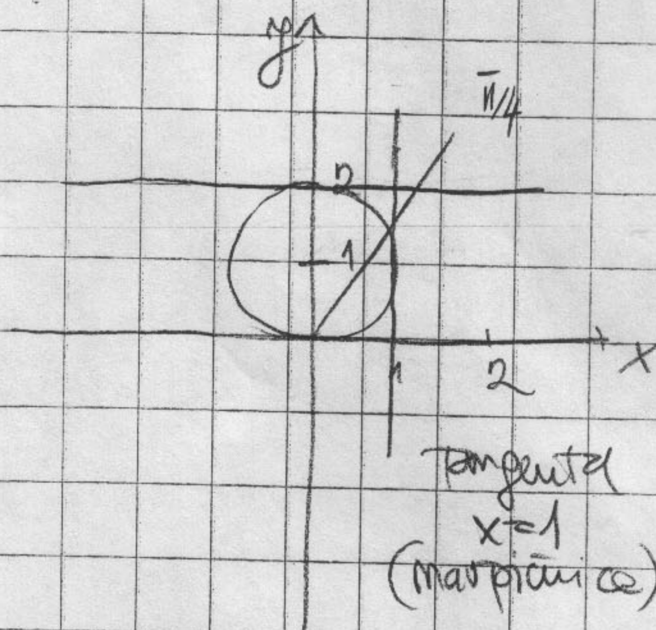
$x_1 = 0$

(2) $k_2 = \frac{y'(\pi/4)}{x'(\pi/4)} = \frac{\sqrt{2}\frac{\sqrt{2}}{2} + 2\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}\sqrt{2} - \sqrt{2}\frac{\sqrt{2}}{2}} = \infty$

(3) $k_3 = \frac{y'(\pi/2)}{x'(\pi/2)} = \frac{0}{-2} = 0$ vodoravnica!!

koo. $x_3 = 0$

$y_3 = 2$ tangenta $\quad y = 2$ premica



tangenta

$x = 1$

(navpičnica)

TAYLORJEVA VRSTA 02.03.2005

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)\, x^n}{n!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \forall\, x \in \mathbb{R}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \qquad \forall\, x \in \mathbb{R}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \qquad \forall\, x \in \mathbb{R}$$

$$\ln(1-x) = \sum_{n=1}^{\infty} \frac{x^n}{n} \qquad x \in (-1, 1)$$

$$\operatorname{sh} x = \frac{e^x - e^{-x}}{2} = \frac{1}{2}\left(e^x - e^{-x}\right)$$

$$= \frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{x^n}{n!} - \sum_{n=0}^{\infty} \frac{(-x)^n}{n!}\right) =$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{x^n}{n!} - \frac{(-x)^n}{n!}\right) =$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \frac{x^n - (-1)^n x^n}{n!} =$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \frac{\left(1 - (-1)^n\right) x^n}{n!} =$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \frac{2\, x^{2n+1}}{(2n+1)!} =$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

$$\boxed{R = \lim_{n\to\infty} \frac{1}{\sqrt[n]{a_n}} \quad \text{ali} \quad R = \lim_{n\to\infty} \frac{a_n}{a_{n+1}}}$$

$$a_n = \frac{1}{(2n+1)!} \qquad R = \lim_{n\to\infty} \frac{\frac{1}{(2n+1)!}}{\frac{1}{(2(n+1)+1)!}} = \lim_{n\to\infty} \frac{(2(n+1)+1)!}{(2n+1)!} = \lim_{n\to\infty} \frac{(2n+3)!}{(2n+1)!} =$$

$$= \lim_{n\to\infty} \frac{(2n+3)(2n+2)\cancel{(2n+1)!}}{\cancel{(2n+1)!}} =$$

$$= \lim_{n\to\infty} (2n+3)(2n+2) = \infty$$

TAYLORJEVA VRSTA FUNKCIJE $\operatorname{sh} x$ KONVERGIRA ZA VSAK $x \in \mathbb{R}$

$0! = 1$
$1! = 1$
$2! = 2$
$3! = 6$

$$\operatorname{ch} x = \frac{e^x + e^{-x}}{2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} \quad \forall\, x \in \mathbb{R}$$

(1.) S POMOČJO RAZVOJA V TAYLORJEVO VRSTO IZRAČUNAJ LIMITO:

a) $$\lim_{x\to 0} \frac{x - \sin x}{x^3} = \lim_{x\to 0} \frac{x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right)}{x^3} =$$

$$= \lim_{x\to 0} \frac{\cancel{x} - \cancel{x} + \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} - \dots}{x^3} \begin{array}{l} /: x^3 \\ /: x^3 \end{array} =$$

$$= \lim_{x\to 0} \left(\frac{1}{3!} - \frac{x^2}{5!} + \frac{x^4}{7!} - \dots \right) = \frac{1}{3!} = \frac{1}{6}$$

b) $$\lim_{x\to 0} \frac{\cos x - 1}{x^2} = \lim_{x\to 0} \frac{\left(\cancel{1} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots\right) - \cancel{1}}{x^2} = \begin{array}{l} /: x^2 \\ /: x^2 \end{array}$$

$$= \lim_{x\to 0} -\frac{1}{2!} + \frac{x^2}{4!} - \frac{x^4}{6!} + \dots =$$

$$= -\frac{1}{2!} = -\frac{1}{2}$$

c) $$\lim_{x\to 0} \frac{6e^x - 6 - 6x - 3x^2 - x^3}{x^4} =$$

$$= \lim_{x\to 0} \frac{6\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots\right) - 6 - 6x - 3x^2 - x^3}{x^4} =$$

$$= \lim_{x\to 0} \frac{\left(\cancel{6} + \cancel{6x} + \cancel{\frac{6x^2}{2!}} + \cancel{\frac{6x^3}{3!}} + \left[\frac{6x^4}{4!} + \dots\right) - \cancel{6} - \cancel{6x} - \cancel{3x^2} - \cancel{x^3}}{x^4} =$$

$$= \lim_{x\to 0} \frac{\frac{6x^4}{4!} + \dots}{x^4} \begin{array}{l} /: x^4 \\ /: x^4 \end{array} =$$

$$= \lim_{x\to 0} \frac{6}{4!} + 6\frac{x}{5!} + 6\frac{x^2}{6!} + 6\frac{x^3}{7!} =$$

$$= \frac{6}{24} = \frac{1}{4}$$

č) $\lim_{x\to 0} \dfrac{6\cdot \ln(1-x)-6x-3x^2-2x^3}{x^4+2x^5+3x^6} =$

$= \lim_{x\to 0} \dfrac{6\left(x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\dots\right)-6x-3x^2-2x^3}{x^4+2x^5+3x^6} =$

$= \lim_{x\to 0} \dfrac{(\cancel{6x}+\cancel{3x^2}+\cancel{2x^3}+\left[\frac{3x^4}{2}+\dots\right)-\cancel{6x}-\cancel{3x^2}-\cancel{2x^3}}{x^4+2x^5+3x^6} = \begin{matrix} /:x^4 \\ /:x^4 \end{matrix}$ DELIMO Z NAJMANJŠO POTENCO, KI JE OSTALA!

$= \lim_{x\to 0} \dfrac{\frac{3}{2}+\frac{6}{5}x+x^2+\dots}{1+\frac{2x^5}{x^4}+\frac{3x^6}{x^4}} =$

$= \lim_{x\to 0} \dfrac{\frac{3}{2}+\frac{6}{5}x+x^2}{1+2x+3x^2} =$ VSTAVIMO $x=0$

$= \dfrac{\frac{3}{2}}{1} = \underline{\underline{\dfrac{3}{2}}}$

d) $\lim_{x\to 0} \dfrac{\operatorname{ch}x - 1}{x\cdot \sin x} =$

$= \lim_{x\to 0} \dfrac{(\cancel{1}+\frac{x^2}{2}+\frac{x^4}{4!}+\frac{x^6}{6!}+\dots)-\cancel{1}}{x\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots\right)} =$

$= \lim_{x\to 0} \dfrac{\frac{x^2}{2}+\frac{x^4}{4!}+\frac{x^6}{6!}+\dots}{x^2-\frac{x^4}{3!}+\frac{x^6}{5!}-\frac{x^8}{7!}+\dots} = \begin{matrix} /:x^2 \\ /:x^2 \end{matrix}$

$= \lim_{x\to 0} \dfrac{\frac{1}{2}+\frac{x^2}{4!}+\frac{x^4}{6!}+\dots}{1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\dots} = \dfrac{\frac{1}{2}}{1} = \underline{\underline{\dfrac{1}{2}}}$

2. $\int \left(x^5+5x^{\frac{3}{2}}+\sqrt{x}+x^{-1}\right)dx =$

$= \int x^5\,dx + \int 5x^{\frac{3}{2}}\,dx + \int \sqrt{x}\,dx + \int x^{-1}\,dx =$

$= \dfrac{x^6}{6} + 5\,\dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + \ln|x| + C$

•) $\int (5x+1)^{10}\,dx = \int t^{10}\,\frac{1}{5}\,dt = \frac{1}{5}\cdot\frac{t^{11}}{11} + C = \dfrac{(5x+1)^{11}}{55} + C$

ODVAJAMO PO x ODVAJAMO PO t

$(5x+1) = t$

$5\,dx = dt$

$\Downarrow$

$dx = \frac{1}{5}\,dt$

$3x=t$ $\qquad$ $7x+1=u$

$3\,dx=dt$ $\qquad$ $7\,dx=du$

$\Downarrow$ $\qquad$ $\Downarrow$

$dx=\frac{1}{3}dt$ $\qquad$ $dx=\frac{du}{7}$

c) $\int\left(\sin(3x)+e^{7x+1}\right)dx =$

$= \int \sin(3x)dx + \int e^{7x+1}dx =$

$= \int \sin t \frac{dt}{3} + \int e^{u}\frac{du}{7} =$

$= -\cos t\cdot\frac{1}{3} + e^{u}\cdot\frac{1}{7} + C =$

$= -\cos(3x)\cdot\frac{1}{3} + e^{7x+1}\cdot\frac{1}{7} + C$

LINEARNI ČLENI !

$5x+1=u$ $\qquad$ $1-x=v$

$5\,dx=du$ $\qquad$ $-dx=dv$

$dx=\frac{1}{5}du$ $\qquad$ $dx=-dv$

č) $\int\left(\frac{1}{5x+1}-\frac{1}{1-x}\right)dx =$

$= \int\frac{dx}{5x+1} - \int\frac{dx}{1-x} =$

$= \int\frac{\frac{1}{5}du}{u} - \int\frac{-dv}{v} =$

$= \frac{1}{5}\ln|u| + \ln|v| + C =$

$= \frac{1}{5}\ln|5x+1| + \ln|1-x| + C =$

d) $\int\frac{dx}{x^2-1} = \int\frac{dx}{(x-1)(x+1)} = *$

$$\frac{1}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)} = \frac{A(x+1)+B(x-1)}{(x+1)(x-1)} =$$

$$= \frac{Ax+A+Bx-B}{(x+1)(x-1)} = \frac{x\underbrace{(A+B)}_{=0}+(A-B)}{(x+1)(x-1)} \quad A-B=1$$

$A+B=0 \quad \Rightarrow \quad A=-B$ $\qquad$ $2A=1$

$A-B=1 \quad \Rightarrow \quad A=1+B$ $\qquad$ $A=\frac{1}{2}$

$B=-\frac{1}{2}$

$x-1=t$ $\qquad$ $x+1=u$

$dx=dt$ $\qquad$ $dx=du$

$* = \int\frac{\frac{1}{2}dx}{x-1} + \int\frac{-\frac{1}{2}dx}{x+1} =$

$= \frac{1}{2}t^{-1}dt + \left(-\frac{1}{2}\right)u^{-1}du =$

$= \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$

e) $\int \frac{x \cdot dx}{x^2+1} =$

$$\boxed{\begin{aligned} x^2+1 &= t \\ 2x\, dx &= dt \end{aligned}}$$

$$x\, dx = \frac{dt}{2}$$

$= \int \frac{\frac{dt}{2}}{t} = \frac{1}{2}\int t^{-1}\, dt =$

$= \frac{1}{2} \cdot \ln|x^2+1| + C$

f) $\int \frac{dx}{x^2+1} = \operatorname{arctg} x + C$

g) $\int \frac{dx}{x \ln x} =$ ŠTEVEC JE ODVOD IMENOVALCA => VZAMEMO NOVO SPREMENLJIVKO

$= \int \frac{dt \cdot \cancel{x}}{\cancel{x} \cdot t} =$

$\ln x = t$

$dt = \frac{1}{x} dx$

$dx = dt \cdot x$

$= \int \frac{dt}{t} =$

$= \ln|t| + C =$

$= \ln|\ln x| + C$

h) $\int \frac{e^x\, dx}{e^x+1} =$

$e^x + 1 = t$

$e^x dx = dt$

$= \int \frac{dt}{t} = \ln|t| + C = \ln|e^x+1| + C$

i) $\int \operatorname{tg} x \cdot dx = \int \frac{\sin x}{\cos x} dx =$

$\cos x = t$

$-\sin x\, dx = dt$

$\sin x\, dx = -\, dt$

$= \int \frac{-dt}{t} = -\ln|t| + C =$

$= -\ln|\cos x| + C$

INTEGRIRANJE PER PARTES

09.03.2005

$$\boxed{\int u\,dv = u\cdot v - \int v\,du}$$

1.) $\int \ln x\,dx =$

$u = \ln x$	$dv = dx$
$du = \frac{1}{x}dx$	$v = x$

$= x\cdot\ln x - \int x\frac{1}{x}dx = \underline{x\ln x - x + C}$

2.) $\int (x^2+x)e^x dx = (x^2+x)e^x - \int e^x(2x+1)dx =$

$u = x^2+x$ (odvod)	$dv = e^x dx$
$du = (2x+1)dx$	$v = e^x$

$u = 2x+1$	$dv = e^x dx$
$du = 2\,dx$	$v = e^x$

$= (x^2+x)e^x - \left((2x+1)e^x - \int 2e^x dx\right) =$

$= (x^2+x)e^x - (2x+1)e^x + 2e^x + C =$

$= \underline{e^x(x^2-x+1) + C}$

$dv \rightarrow$ vzamemo tako funkcijo, ki jo znamo integrirat!

3.) $\int \arcsin x\,dx = x\cdot\arcsin x - \int x\frac{1}{\sqrt{1-x^2}}dx =$

$u = \arcsin x \qquad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}dx \qquad v = x$

ČE V ŠTEVCU NASTOPA IZRAZ, KI JE ODVOD IMENOVALCA, UPORABIMO NOVO SPREMENLJIVKO, DA SE BO KASNEJE ŠTEVEC LAHKO OKRAJŠAL.

$t = 1-x^2$

$dt = -2x\,dx \Rightarrow dx = -\frac{dt}{2x}$

$= x\cdot\arcsin x + \int \frac{\cancel{x}}{\sqrt{t}}\cdot\frac{dt}{2\cancel{x}} =$

$= x\cdot\arcsin x + \frac{t^{\frac{1}{2}}}{\frac{1}{2}}\cdot\frac{1}{2} + C =$

$= x\cdot\arcsin x + \quad\cdot\frac{1}{2} + C =$

$= x\cdot\arcsin x + \sqrt{1-x^2} + C =$

4.) $\int x^2 \operatorname{arctg} x\, dx = \operatorname{arctg} x \cdot \frac{1}{3}x^3 - \int \frac{x^3}{3} \cdot \frac{1}{1+x^2}\, dx =$

$u = \operatorname{arctg} x \qquad dv = x^2\, dx$

$du = \frac{1}{1+x^2}\, dx \qquad v = \frac{1}{3}x^3$

$t = 1+x^2 \quad \Rightarrow x^2 = t-1$

$dt = 2x\, dx$

$\Rightarrow dx = \frac{dt}{2x}$

$= \operatorname{arctg} \frac{x^3}{3} - \int \frac{x^{\cancel{3}\,2}}{3} \cdot \frac{1}{t} \cdot \frac{dt}{2\cancel{x}} =$

$= \operatorname{arctg} \frac{x^3}{3} - \int \frac{t-1}{3} \cdot \frac{1}{t} \cdot \frac{dt}{2} =$

$= \operatorname{arctg} \frac{x^3}{3} - \frac{1}{6}\int \frac{t}{t} - \frac{1}{t} \cdot dt =$

$= \operatorname{arctg} \frac{x^3}{3} - \frac{1}{6}\int 1 - \frac{1}{t}\, dt =$

$= \operatorname{arctg} \frac{x^3}{3} - \frac{1}{6}\left(t - \ln|t|\right) + C =$

KO INTEGRIRAMO VSE, DAMO NOT VREDNOST [illegible] SPREMENLJIVKE!

$= \operatorname{arctg} \frac{x^3}{3} - \frac{1}{6}\left(1+x^2 - \ln|1+x^2|\right) + C$

5.) $\int \cos x\, e^x\, dx = e^x \cos x - \int e^x(-\sin x)\, dx =$

$= e^x \cos x + \int e^x \sin x\, dx =$

$= e^x \cos x + e^x \sin x - \int e^x \cos x\, dx =$

① $u = \cos x$

$du = -\sin x\, dx$

$dv = e^x\, dx$

$v = e^x$

② $u = \sin x$

$du = \cos x$

$dv = e^x\, dx$

$v = e^x$

$2\int \cos x\, e^x\, dx = e^x \cos x + e^x \sin x + 2C$

$\int \cos x\, e^x\, dx = \frac{\cos x + \sin x}{2} e^x + C$

6.) $\int \frac{dx}{2x^2+3x-2} = \int \frac{dx}{(2x-1)(x+2)} = **$

RAZSTAVIMO IMENOVALEC

① $\frac{1}{(2x-1)(x+2)} = \frac{A}{(2x-1)} + \frac{B}{(x+2)} = \frac{Ax + A2 + B\cdot 2x - B}{(2x-1)(x+2)} =$

$= \frac{x(A+2B) + 2A - B}{(2x-1)(x+2)}$

KER V ŠTEVCU NI NOBENIH X-OV JE $A+2B=0$

IN KER IMA KONSTANTO 1 JE $2AB=1$

$A + 2B = 0$

$2A - B = 1$

$-4B - B = 1 \quad \Rightarrow \quad B = -\frac{1}{5}$

$\Rightarrow A = -2B = \frac{2}{5}$

$$** = \int \left(\frac{\frac{2}{5}}{2x-1} + \frac{-\frac{1}{5}}{x+2} \right) dx =$$

$$= \frac{2}{5} \int \frac{dx}{2x-1} - \frac{1}{5} \int \frac{dx}{x+2} = \begin{bmatrix} 2x-1=t & x+2=s \\ 2\,dx=dt & dx=ds \end{bmatrix}$$

$$= \frac{2}{5} \int \frac{dt}{\cancel{2}t} - \frac{1}{5} \int \frac{ds}{s} =$$

$$= \frac{1}{5} \ln|t| - \frac{1}{5} \ln|s| + C =$$

$$= \frac{1}{5} \ln|2x-1| - \frac{1}{5} \ln|x+2| + C$$

7.) $$\int \frac{x^2\,dx}{x^2-x-2} = \int \frac{x^2\,dx}{(x+1)(x-2)} =$$

PREDEN DAMO NA PARCIALNE ULOMKE, SE ZNEBIMO x^2 (DELIMO)!
~~ČE~~ STOPNJA ŠTEVCA MORA BITI STROGO MANJŠA OD ST. IMENOVALCA!

$$x^2 : (x^2-x-2) = 1$$
$$\underline{x^2-x-2}$$
$$x+2$$

$$= \int 1 + \frac{x+2}{(x+1)(x-2)} = \int \left(1 - \frac{-\frac{1}{3}}{(x+1)} + \frac{\frac{4}{3}}{(x-2)} \right) dx =$$

$$\frac{x+2}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} =$$

$$= \frac{A(x-2) + B(x+1)}{(x+1)(x-2)} =$$

IZPOSTAVIMO VSE x-e $$= \frac{Ax - 2A + Bx + B}{(x+1)(x-2)} =$$

$$= \frac{x(A+B) - 2A + B}{(x+1)(x-2)}$$

$$A+B = 1 \Rightarrow A = 1-B$$
$$\underline{-2A+B = 2} \qquad A = 1 - \frac{4}{3} = -\frac{1}{3}$$
$$-2(1-B) + B = 2$$
$$-2 + 2B + B = 2$$
$$3B = 4$$
$$B = \frac{4}{3}$$

$$= \int 1\,dx + \int \frac{-\frac{1}{3}}{(x+1)}\,dx + \int \frac{\frac{4}{3}}{(x-2)}\,dx =$$

$$= x + \left(-\frac{1}{3}\right) \int \frac{dt}{t} + \frac{4}{3} \int \frac{dz}{z} =$$

$$\begin{bmatrix} t = x+1 & z = x-2 \\ dt = 1\,dx & dz = 1\,dx \end{bmatrix}$$

$$= x - \frac{1}{3} \ln|t| + \frac{4}{3} \ln|z| + C$$

$$= x - \frac{1}{3} \ln|x+1| + \frac{4}{3} \ln|x-2| + C$$

$$\frac{x}{a} = t$$
$$\frac{dx}{a} = dt \Rightarrow dx = a\,dt$$

★ $$\int \frac{dx}{x^2+a^2} = \int \frac{dx}{a^2\left(\frac{x^2}{a^2}+1\right)} = \frac{1}{a^2}\int \frac{dx}{\left(\frac{x}{a}\right)^2+1} = \frac{1}{a^{\cancel{2}}}\int \frac{\cancel{a}\,dt}{t^2+1} =$$
$$= \frac{1}{a} \operatorname{arctg} t + C = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$$

★ $$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$$

$$a^2 = 4 \Rightarrow a = 2$$

8.) $$\int \frac{dx}{x^2+2x+5} = \int \frac{dx}{(x+1)^2+4} = \int \frac{dt}{t^2+4} = \frac{1}{2} \operatorname{arctg} \frac{t}{2} + C =$$

DAMO NA POPOLNI KVADRAT

$$= \frac{1}{2} \operatorname{arctg} \frac{x+1}{2} + C$$

$D = b^2 - 4ac$

$D = 4 - 20 = -6$

$x+1 = t$

$dx = dt$

9.) $$\int \frac{x^2+x+1}{x^2+2}\,dx = \int 1\,dx + \int \frac{x-1}{x^2+2}\,dx =$$
$$= \int 1\,dx + \int \frac{x}{x^2+2}\,dx + \int \frac{-1}{x^2+2}\,dx =$$
$$= x + \int \frac{\cancel{x}}{t}\frac{dt}{2\cancel{x}} - \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x}{\sqrt{2}} =$$
$$= x + \int \frac{1}{2}\frac{dt}{t} - \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x}{\sqrt{2}} =$$
$$= x + \frac{1}{2} \ln|t| - \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x}{\sqrt{2}} + C =$$
$$= x + \frac{1}{2} |x^2+2| - \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x}{\sqrt{2}} + C$$

$(x^2+x+1) : (x^2+2) = 1$
$-(x^2+2)$
$x-1$

$t = x^2+2 \Rightarrow dt = 2x\,dx$

$dx = \frac{dt}{2x}$

9) $$\int \frac{x+1}{x^2+6x+10}\,dx = \int \frac{x+1}{(x+3)^2+1}\,dx = \int \frac{x}{(x+3)^2+1}\,dx + \int \frac{1}{(x+3)^2+1}\,dx =$$

$$\boxed{\begin{aligned} t &= x^2+6x+10 \\ dt &= 2x+6\;dx \\ dt &= 2(x+3)\,dx \\ dx &= \frac{dt}{2x+6} \end{aligned}}$$

$$= \int \frac{x+3}{(x+3)^2+1}\,dx + \int \frac{1-3}{(x+3)^2+1}\,dx =$$
$$= \int \frac{\cancel{(x+3)}\,dt}{2\cancel{(x+3)}\,t} + (-2)\frac{1}{1} \operatorname{arctg}(x+3) =$$
$$= \frac{1}{2} \ln|x^2+6x+10| - 2 \operatorname{arctg}(x+3) + C$$

10.) $\int \frac{x+1}{x^3+x}\,dx = \int \frac{x+1}{x(x^2+1)} = *$

$\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ ŠTEVEC MORA BITI ZA 1 STOPNJO NIŽJI KOT IMENOVALEC

$= \frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)} =$

$= \frac{x^2(A+B)+Cx+A}{x(x^2+1)}$

PRIMERJAMO KOEFICIENTE:

x^2 : $A+B=0$ x^2 NE NASTOPA V ŠTEVCU $(x+1)$ $\Rightarrow B=-A$

x : $C=1$ $B=-1$

1 : $A=1$

$$\left[\begin{array}{l} x^2+1=t \\ 2x\,dx=dt \\ \quad \hookrightarrow dx=\frac{dt}{2x} \end{array}\right]$$

$* = \int\left(\frac{1}{x} + \frac{-x+1}{x^2+1}\right)dx = \int \frac{1}{x}\,dx + \int \frac{-x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx =$

$= \ln|x| + \int \frac{-\cancel{x}}{t}\frac{dt}{2\cancel{x}} + \operatorname{arctg} x =$

$= \ln|x| + \frac{1}{2}\ln|t| + \operatorname{arctg} x + C =$

$= \ln|x| + \frac{1}{2}\ln|x^2+1| + \operatorname{arctg} x + C$

$\boxed{A=1}$ $A+B=0$ $-2A-B+C=0$

$B=-A$ $-2-(-1)+C=0$

$\boxed{B=-1}$ $\boxed{C=1}$

$$e^x = t$$
$$dx\, e^x = dt$$
$$dx = \frac{dt}{e^x}$$

15.03.2004

1.) $$\int \frac{e^{2x}dx}{e^{2x}-3e^x-10} = \int \frac{t^2 \frac{dt}{e^x}}{t^2-3t-10} = \int$$

$$= \int \frac{t\ dt}{[(t-5)(t+2)]} = \int \frac{\frac{5}{7}}{(t-5)} + \frac{\frac{2}{7}}{(t+2)}\ dt = \frac{5}{7}\int \frac{1}{t-5}dt + \frac{2}{7}\int \frac{1}{t+2}\ dt =$$

$$\frac{t}{(t-5)(t+2)} = \frac{A}{t-5} + \frac{B}{t+2} = \frac{A(t+2)+B(t-5)}{(t-5)(t+2)} =$$

$$= \frac{At+2A+Bt-5B}{(t-5)(t+2)} = \frac{t(A+B)+2A-5B}{(t-5)(t+2)}$$

$$A+B=1 \Rightarrow [A = 1-B = 1-\tfrac{2}{7} = \tfrac{5}{7}]$$
$$2A-5B=0$$
$$2(1-B)-5B=0$$
$$\boxed{B=\tfrac{2}{7}}$$

$$= \frac{5}{7}\ln(t-5) + \frac{2}{7}\ln(t+2) + C$$
$$= \frac{5}{7}\ln(e^x-5) + \frac{2}{7}\ln(e^x+2) + C$$

$$\boxed{(a^x)' = a^x \cdot \ln a}$$

2.) $$\int \frac{dx}{4^x - 2\cdot 2^x + 1} = \int \frac{\frac{dt}{2^x \ln 2}}{t^2-2t+1} = \int \frac{\frac{dt}{t\cdot \ln 2}}{(t-1)(t-1)} =$$

$4^x \rightarrow (2)^{2x}$

$$\left[\begin{array}{l} 2^x = t \\ dx\ 2^x \ln 2 = dt \end{array}\right]$$

$$= \int \frac{dt}{[t\cdot \ln 2\,(t-1)(t-1)]} = \left(\int \frac{dt}{t} - \int \frac{1\,dt}{t-1} + \int \frac{dt}{(t-1)^2}\right)\frac{1}{\ln 2} =$$

TO JE NEKA KONSTANTA

$$= \left(\ln|t| - \ln|t-1| + \frac{(t-1)^{-1}}{-1}\right)\frac{1}{\ln 2} + C =$$
$$= \left(\ln|2^x| - \ln|2^x-1| + \frac{(2^x-1)^{-1}}{-1}\right)\frac{1}{\ln 2} + C$$

$$\frac{1}{t(t-1)(t-1)} = \frac{A}{t} + \frac{B}{(t-1)} + \frac{C}{(t-1)^2} =$$

$$= \frac{A(t-1)^2 + Bt(t-1) + Ct}{t(t-1)^2} =$$

$$= \frac{At^2 - 2At + A + Bt^2 - Bt + Ct}{t(t-1)^2} =$$

$$= \frac{t^2(A+B) + t(-2A-B+C) + A}{t(t-1)^2}$$

$$\boxed{\int \frac{1}{x^2}dx = \int x^{-2}dx = \frac{x^{-1}}{-1} + C}$$

3.) $\int \frac{\sin x \, dx}{\cos^2 x \cos x} = \int \frac{\cancel{\sin x}\left(-\frac{dt}{\cancel{\sin x}}\right)}{t^2 - t} = -\int \frac{dt}{[t^2 - t]} =$

$\cos x = t$

$-dx \sin x = dt$

$dx = \frac{dt}{-\sin x}$

$= -\left(\int -\frac{1\,dt}{t} + \int \frac{dt}{t-1}\right) =$

$= \ln|t| - \ln|t-1| + C =$

$= \ln|\cos x| - \ln|\cos x - 1| + C$

$\frac{1}{t^2 - t} = \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} =$

$= \frac{A(t-1) + Bt}{t(t-1)} = \frac{At - A + Bt}{t(t-1)} =$

$= \frac{t(A+B) - A}{t(t-1)}$

$A + B = 0 \qquad -A = 1$

$A = -B \qquad \boxed{A = -1}$

$\boxed{B = 1}$

4.) $\int \frac{\sin 2x \, dx}{\sin^3 x + 2\sin x} = \int \frac{2\cancel{\sin x}\cos x \, dx}{\cancel{\sin x}(\sin^2 x + 2)} = \int \frac{2\cos x \, dx}{\sin^2 x + 2} =$

$\sin x = t$

$\cos x \, dx = dt$

$dx = \frac{dt}{\cos x}$

$= \int \frac{2\cancel{\cos x}\,\frac{dt}{\cancel{\cos x}}}{t^2 + 2} = \int \frac{2\,dt}{t^2+2} = 2\cdot\left(\frac{1}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}}\right) + C =$

$2 \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{2}\cdot 2}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{2}\cdot\cancel{2}}{\cancel{2}} = \sqrt{2}$

$= \sqrt{2} \operatorname{arctg} \frac{\cos x}{\sqrt{2}} + C$

$$\boxed{\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C}$$

$$\int \frac{dx}{\sqrt{x^2+A}} = \ln\left|x+\sqrt{\quad +A}\right| + C$$

5.) $\displaystyle\int \frac{dx}{\sqrt{x^2+2x}} = \int \frac{dx}{\sqrt{(x+1)^2-1}} = \int \frac{dt}{\sqrt{t^2-1}} = \ln\left|t+\sqrt{t^2-1}\right| + C =$

$x+1=t$

$dx=dt$

$$= \underline{\ln\left|x+1+\sqrt{(x+1)^2-1}\right| + C}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$$

6.) $\displaystyle\int \frac{dx}{\sqrt{3+2x-x^2}} = \int \frac{dx}{\sqrt{-(x\ 1)^2+4}} =$

$x-1=t$

$dx=dt$

$$= \int \frac{dt}{\sqrt{-(t)^2-4}} = \arcsin\frac{t}{2} + C = \underline{\underline{\arcsin\frac{x-1}{2} + C}}$$

INTEGRACIJA TRIGONOMETRIČNIH FUNKCIJ

$$\int \sin^n x \cdot \cos^m x \, dx$$

1.1.) če n je liho število $\quad t = \cos x$

1.2.) če m je liho število $\quad t = \sin x$

1.2.3.) če m in n sodo število

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

1.) $\displaystyle\int \sin^8 x \cdot \cos^3 x \, dx = \int t^8 \cos^{\cancel{3}} x \cdot \frac{dt}{\cancel{\cos x}} = \int t^8 \cos^2 x \, dt =$

$t=\sin x$

$dt = \cos x \, dx$

$dx = \frac{dt}{\cos x}$

$$= \int t^8 (1-\sin^2 x)\, dt = \int t^8 (1-t^2)\, dt =$$

$$= \int (t^8 - t^{10})\, dt = \int t^8\, dt - \int t^{10}\, dt =$$

$$= \frac{t^9}{9} - \frac{t^{11}}{11} + C = \underline{\underline{\frac{\sin^9 x}{9} - \frac{\sin^{11} x}{11} + C}}$$

$$2.)\quad \int \sin^5 4x\cdot\cos^2 4x\,dx = \int \cancel{\sin^5 4x}^{4}\cdot t^2\,\frac{dt}{-4\cancel{\sin 4x}} =$$

$\cos 4x = t$

$-4\sin 4x\,dx = dt$

$dx = \frac{dt}{-4\sin 4x}$

$$= -\frac{1}{4}\int \sin^4 4x\cdot t^2\,dt =$$

$$= -\frac{1}{4}\int (1-\cos^2 4x)^2\cdot t^2\,dt =$$

$$= -\frac{1}{4}\int (1-t^2)^2\cdot t^2\,dt =$$

$$= -\frac{1}{4}\int (1-2t^2+t^4)t^2\,dt =$$

$$= -\frac{1}{4}\int t^2-2t^4+t^6\,dt =$$

$$= -\frac{1}{4}\left(\frac{t^3}{3}-\frac{2t^5}{5}+\frac{t^7}{7}\right)+C = -\frac{1}{4}\left(\frac{\cos^3 4x}{3}-\frac{2\cos^5 4x}{5}+\frac{\cos^7 4x}{7}\right)+C$$

$$3.)\quad \int \sin^4 x\cdot\cos^2 x\,dx = \int\left(\frac{1-\cos 2x}{2}\right)^2\left(\frac{1+\cos 2x}{2}\right)dx =$$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$$= \int \frac{1-2\cos 2x+\cos^2 2x}{2}\,\frac{1+\cos 2x}{2}\,dx =$$

$$= \int \frac{(1-2\cos 2x+\cos^2 2x)(1+\cos 2x)}{4}\,dx =$$

$$= \int \frac{1-\cancel{2}\cos 2x+\cos^2 2x+\cos 2x-\cancel{2}\cos^2 2x+\cos^3 2x}{\cancel{4}}\,dx$$

$$= \int \frac{1-\cos 2x-\cos^2 2x+\cos^3 2x}{8}\,dx =$$

$$= \int \frac{1}{8}\,dx - \int\frac{\cos 2x}{8}\,dx - \int\frac{\cos^2 2x}{8}\,dx + \int\frac{\cos^3 2x}{8}\,dx = *$$

$2x = t$

$t = \sin 2x$

$$\int \frac{\cos^2 2x}{8} = \int \frac{\frac{1+\cos 2x}{2}}{8}\,dx = \int\frac{1+\cos 4x}{16}\,dx = \frac{x}{16}+\int\frac{\cos 4x}{16}\,dx =$$

$$= \frac{x}{16}+\int\frac{\cos t}{16}\cdot\frac{dt}{4} = \frac{x}{16}\int\frac{\cos t}{64}\,dt$$

$t = 4x$

$dt = 4dx$

$dx = \frac{dt}{4}$

$$= \frac{x}{16}+\frac{1}{64}\sin t + C = \frac{x}{16}+\frac{1}{64}\sin 4x + C$$

$\int R(\sin x, \cos x)\,dx$; R JE RACIONALNA FUNKCIJA

$\Rightarrow$ NOVA SPREMENLJIVKA JE $\boxed{t = \operatorname{tg}\frac{x}{2}}$

$$\operatorname{tg}\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = t$$

$$\sin\frac{x}{2} = t\cos\frac{x}{2} \quad \Big|^2$$

$$\sin^2\frac{x}{2} = t^2\cos^2\frac{x}{2}$$

$$1 - \cos^2\frac{x}{2} = t^2\cos^2\frac{x}{2}$$

$$1 = t^2\cos^2\frac{x}{2} + \cos^2\frac{x}{2}$$

$$\cos^2\frac{x}{2} = \frac{1}{1+t^2}$$

$$\sin^2\frac{x}{2} = 1 - \cos^2\frac{x}{2} = 1 - \frac{1}{1+t^2} + \frac{t^2}{1+t^2}$$

$$\cos x = \cos 2\frac{x}{2} = \cos^2\frac{x}{2} - $$

$$= \frac{1}{1+t^2} - \frac{t^2}{1+t^2}$$

23.03.2005

1.) $$\int \frac{dx}{1+\sin x+\cos x} = \int \frac{\frac{2dt}{1+t^2}}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}} = \int \frac{\frac{2dt}{\cancel{1+t^2}}}{\frac{1+t^2+2t+1-t^2}{\cancel{1+t^2}}} =$$

$t = tg\frac{x}{2}$

$$= \int \frac{2dt}{2+2t} = \int \frac{\cancel{2}dt}{\cancel{2}(1+t)} =$$

$$= \int \frac{dt}{1+t} = \ln|1+t| + C =$$

$$= \ln\left|tg\frac{x}{2}+1\right| + C$$

2.) $$\int \frac{\cos x\, dx}{1+\cos x} = \int \frac{\frac{1-t^2}{1+t^2}\frac{2dt}{1+t^2}}{1+\frac{1-t^2}{1+t^2}} = \int \frac{\frac{2dt\,(1-t^2)}{(1+t^2)^{\cancel{2}}}}{\frac{1+t^2+1-t^2}{\cancel{1+t^2}}} =$$

$t = tg\frac{x}{2}$

$$= \int \frac{\frac{\cancel{2}dt\,(1-t^2)}{1+t^2}}{\cancel{2}} = \int \frac{(1-t)^2 dt}{1+t^2} =$$

STOPNJA ŠTEVCA MORA BITI ENAKA ALI MANJŠA OD IMENOVALCA

DELIMO!

$$(1-t)^2 : (1+t^2) = -1$$
$$\underline{-1-t^2}$$
$$2$$

$$= \int \left(-1+\frac{2}{1+t^2}\right)dt = \int -dt + \int \frac{2dt}{1+t^2} =$$

$$= -t + 2\,\mathrm{arctg}\,t + C =$$

$$= -tg\frac{x}{2} + 2\,\mathrm{arctg}\left(tg\frac{x}{2}\right) + C$$

3.) $$\int \frac{1+tg x}{1-tg x}dx = \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}dx = \int \frac{\frac{\cos x+\sin x}{\cancel{\cos x}}}{\frac{\cos x-\sin x}{\cancel{\cos x}}} * dx =$$

1. NAČIN

$t = tg\frac{x}{2}$

$$= \int \frac{\frac{1-t^2}{\cancel{1+t^2}}+\frac{2t}{\cancel{1+t^2}}}{\frac{1-t^2}{\cancel{1+t^2}}-\frac{2t}{\cancel{1+t^2}}}\frac{2dt}{1+t^2} = \int \frac{1-t^2+2t}{1-t^2-2t}\frac{2dt}{1+t^2} =$$

$$D = 4+4 = 8 \qquad t_{1,2} = \frac{2+2\sqrt{2}}{-2} \to t_1 = -1-\sqrt{2}, \quad t_2 = -1+\sqrt{2}$$

$$= \int \frac{-t^2+2t+1}{-(t-(-1-\sqrt{2}))(t-(-1+\sqrt{2}))}\cdot\frac{2\,dt}{1+t^2} =$$

RAZSTAVIMO NA PARCIALNE ULOMKE

za domačo nalogo

$t = \operatorname{tg}\frac{x}{2}$	$t = \operatorname{tg} x$
$\cos x = \frac{1-t^2}{1+t^2}$	$\cos x = \frac{1}{\sqrt{1+t^2}}$
$\sin x = \frac{2t}{1+t^2}$	$\sin x = \frac{t}{\sqrt{1+t^2}}$
$dx = \frac{2dt}{1+t^2}$	$dx = \frac{dt}{1+t^2}$

2. NAČIN

$u = \cos x - \sin x$

$du = -\sin x - \cos x \, dx$

$dx = \frac{du}{-\sin x - \cos x}$

$$* = \int \frac{\cos x + \sin x}{\cos x - \sin x} = \int \frac{\cancel{\cos x + \sin x}}{u} \frac{du}{-(\cancel{\sin x + \cos x})} =$$

$$= -\int u^{-1}\, du = -\ln|u| = -\ln|\cos x - \sin x| + C$$

4.) $$\int \frac{dx}{3\sin^2 x + 5\cos^2 x} = \int \frac{\frac{dt}{1+t^2}}{3\left(\frac{t}{\sqrt{1+t^2}}\right)^2 + 5\left(\frac{1}{\sqrt{1+t^2}}\right)^2} = \int \frac{dt}{3t^2+5} = \int \frac{dt}{3\left(t^2+\frac{5}{3}\right)} =$$

$t = \operatorname{tg} x$

$$= \frac{1}{3} \frac{1}{\sqrt{\frac{5}{3}}} \operatorname{arctg} \frac{t}{\sqrt{\frac{5}{3}}} + C = \frac{1}{3} \cdot \frac{1}{\sqrt{\frac{5}{3}}} \operatorname{arctg} \frac{\operatorname{tg} x}{\sqrt{5/3}} + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$$

1. NAČIN $t = \operatorname{tg} x$

5.) $$\int \frac{\sin 2x \, dx}{1+\sin^2 x} = \int \frac{2\sin x \cos x \, dx}{1+\sin^2 x} \overset{*}{=} \int \frac{2 \frac{t}{\sqrt{1+t^2}} \cdot \frac{1}{\sqrt{1+t^2}} \frac{dt}{1+t^2}}{1+\left(\frac{t}{\sqrt{1+t^2}}\right)^2} =$$

$$= \int \frac{\frac{2t\, dt}{(1+t^2)(1+t^2)}}{\frac{1+t^2+t^2}{(1+t^2)}} = \int \frac{2t\, dt}{(1+t^2)(1+2t^2)} = \dots$$

(RAČUN 7.)
KOT PRI PREJŠNJI NALOGI RAZCEPIMO NA PARCIALNE ULOMKE!

2. NAČIN

* $u = \sin x$

$du = \cos x \, dx$

$dx = \frac{du}{\cos x}$

$$= \int \frac{2\sin x \cos x \, dx}{1+\sin^2 x} = \int \frac{2u\, du}{1+u^2} = \int \frac{dv}{v} =$$

$v = 1+u^2$

$dv = 2u\, du$

$$= \ln|v| + C = \ln|1+u^2| + C = \ln|1+\sin^2 x| + C$$

$\rightarrow t = \arcsin x$

$x = \sin t$
$dx = \cos t\, dt$

$$\boxed{1-(\sin t)^2 = (\cos t)^2}$$

6.) $$\int \frac{dx}{(1+x^2)\cdot\sqrt{1-x^2}} = \int \frac{\cos t\, dt}{(1+\sin t^2)\cdot \boxed{\sqrt{1-\sin^2 t}}} =$$

$$= \int \frac{\cos t\, dt}{(1+\sin^2 t)\sqrt{\cos^2 t}} = \int \frac{\cancel{\cos t}\, dt}{1+\sin^2 t\cdot \cancel{\cos t}} = \int \frac{dt}{1+\sin^2 t} = \quad s = \operatorname{tg} x$$

$$= \int \frac{\frac{ds}{1+s^2}}{1+\left(\frac{s}{\sqrt{1+s^2}}\right)^2} = \int \frac{\frac{ds}{\cancel{(1+s^2)}}}{\frac{1+s^2+s^2}{\cancel{(1+s^2)}}} = \int \frac{ds}{1+s^2+s^2} = \int \frac{ds}{1+2s^2} =$$

$$= \int \frac{ds}{2(\frac{1}{2}+s^2)} = \frac{1}{2}\frac{1}{\sqrt{\frac{1}{2}}}\operatorname{arctg}\frac{s}{\sqrt{\frac{1}{2}}} + C = \frac{1}{2}\frac{1}{\sqrt{\frac{1}{2}}}\operatorname{arctg}\frac{\operatorname{tg} t}{\sqrt{\frac{1}{2}}} + C =$$

$$= \frac{1}{2}\frac{1}{\sqrt{1/2}}\operatorname{arctg}\frac{\operatorname{tg}\arcsin x}{\sqrt{1/2}} + C = \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{\sqrt{2}x}{\sqrt{1-x^2}} + C$$

$x = \sin t$
$x^2 = \sin^2 t - 1 - \cos^2 t$
$\cos^2 t = 1 - x^2$
$\cos t = \sqrt{1-x^2}$
$\operatorname{tg} t = \frac{x}{\sqrt{1-x^2}}$

7.) $$\int \frac{\sin^2 x\, dx}{1+\cos^2 x} = \int \frac{\left(\frac{t}{\sqrt{1+t^2}}\right)^2 \frac{dt}{1+t^2}}{1+\left(\frac{1}{\sqrt{1+t^2}}\right)^2} = \int \frac{\frac{t^2}{\cancel{1+t^2}}\cdot\frac{dt}{1+t^2}}{\frac{1+t^2+1}{\cancel{1+t^2}}} =$$

$$= \int \frac{t^2\, dt}{(1+t^2+1)(1+t^2)} = \int \frac{t^2\, dt}{(t^2+2)(1+t^2)} =$$

KER JE II. STOPNJE!

$$\frac{t^2}{(t^2+2)(1+t^2)} = \frac{At+B}{(t^2+2)} + \frac{Ct+D}{1+t^2} = \frac{At^3+Bt^2+At+B+Ct^3+2Ct+Dt^2+2D}{(t^2+2)(t^2+1)} =$$

$$= \frac{t^3(A+C) + t^2(B+D) + t(A+2C) + B + 2D}{(t^2+2)(t^2+1)} =$$

$A+C=0 \qquad B+D=1 \qquad A+2C=0 \qquad B+2D=0$

$A=-C \qquad B=1-D \qquad -C+2C=0 \qquad 1-D+2D=0$

$C=0 \qquad 1+D=0$

$D=-1$

$$= \int \frac{2}{t^2+2}\,dt + \int \frac{-1}{t^2+1}\,dt =$$

$$= 2\frac{1}{\sqrt2}\,\mathrm{arctg}\,\frac{t}{\sqrt2} - \mathrm{arctg}\,t + C =$$

$$= 2\cdot\frac{1}{\sqrt2}\,\mathrm{arctg}\,\frac{\mathrm{tg}x}{\sqrt2} - \mathrm{arctg}(\mathrm{tg}x) + C$$

30.03.2005

1. $$\int \frac{e^{3x}+e^x+1}{e^{2x}+1}\,dx = \int \frac{t^3+t+1}{t^2+1}\,\frac{dt}{t} = \int \frac{t^3+t+1}{t(t^2+1)}\,dt =$$

$e^x = t$

$e^x dx = dt$

$dx = \frac{dt}{e^x} = \frac{dt}{t}$

$(t^3+t+1):(t^3+t) = 1$

t^3+t

$\boxed{1}$

$$= \int \left(1 + \frac{\boxed{1}}{t(t^2+1)}\right)dt = \int \left(1+\frac{1}{t}-\frac{t}{t^2+1}\right)dt =$$

$$\frac{1}{t(t^2+1)} = \frac{A}{t} + \frac{Bt+C}{t^2+1} = \frac{At^2+A+Bt^2+Ct}{t(t^2+1)}$$

$t^2: \quad A+B=0 \rightarrow B=-1$

$t: \quad C=0$

$1: \quad A=1$

$u=t^2+1$

$du = 2t\,dt$

$t\,dt = \frac{1}{2}du$

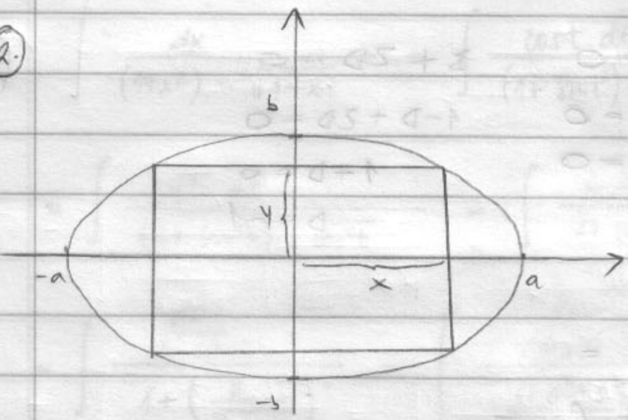
$$= \int dt + \int \frac{dt}{t} - \int \frac{t\,dt}{t^2+1} = t + \ln|t| - \int \frac{\frac12 du}{u} =$$

$$= t + \ln|t| - \frac12 \ln(t^2+1) + C =$$

$$= e^x + \ln e^x - \frac12 \ln(t^2+1) + C =$$

$$= e^x + x - \frac12 \ln(e^{2x}+1) + C$$

2.



$$p = (2x)(2y)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y = +b\sqrt{1-\frac{x^2}{a^2}}$$

$$p(x) = 4xb\sqrt{1-\frac{x^2}{b^2}}$$

$$p'(x) = 4b\sqrt{1-\frac{x^2}{a^2}} + 4xb\frac{1}{2}\left(1-\frac{x^2}{a^2}\right)^{-\frac{1}{2}}\left(-\frac{2x}{a^2}\right) =$$

$$= 4b\sqrt{1-\frac{x^2}{a^2}} - \frac{4x^2b}{a^2\sqrt{1-\frac{x^2}{a^2}}} = 0$$

$$4b\sqrt{1-\frac{x^2}{a^2}} = \frac{4a^2b}{a^2\sqrt{1-\frac{x^2}{a^2}}} \quad / \cdot a^2\sqrt{1-\frac{x^2}{a^2}}$$

$$4a^2b\left(1-\frac{x^2}{a^2}\right) = 4x^2b$$

$$4a^2b - 4bx^2 = 4bx^2$$

$$4a^2b = 8bx^2 \quad /: 4b$$

$$a^2 = 2x^2$$

$$x = \frac{a}{\sqrt{2}}$$

$$y = b\sqrt{1-\frac{x^2}{2a^2}} = \frac{b}{\sqrt{2}}$$

STRANICI PRAVOKOTNIKA STA $\sqrt{2}a$ IN $\sqrt{2}b$,
PLOŠČINA PA $p = 2ab$

$$2\frac{a}{\sqrt{2}} \cdot \cancel{\frac{a}{\sqrt{2}}} =$$

$$\frac{2a\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\cancel{2}a\sqrt{2}}{\cancel{2}}$$

$$\sqrt{1-x^2} \Rightarrow x = \sin t$$

$$\sqrt{1+x^2} \Rightarrow x = \operatorname{tg} t$$

$1+\operatorname{tg}^2 t - 1 + \frac{\sin^2 t}{\cos^2 t} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$

3. $$\int \frac{dx}{(1-x^2)\sqrt{1+x^2}} = \int \frac{dt}{\cos^2 t\,(1-\operatorname{tg}^2 t)\sqrt{\frac{1}{\cos^2 t}}} =$$

$$= \int \frac{dt}{\cos^2 t\left(1-\frac{\sin^2 t}{\cos^2 t}\right)\frac{1}{\cos t}} = \int \frac{\cos t\, dt}{\cos^2 t - \sin^2 t} =$$

$$= \int \frac{\cos t\, dt}{1-\sin^2 t - \sin^2 t} = \int \frac{\cos t\, dt}{1-2\sin^2 t} = \int \frac{du}{1-2u^2} =$$

$$\left[\begin{array}{l} u = \sin t \\ du = \cos t\, dt \end{array}\right]$$

rn. parcialni ulomki!

$$= \int \frac{du}{(1-\sqrt{2}u)(1+\sqrt{2}u)} = \int \left(\frac{\frac{1}{2}}{1-\sqrt{2}u} + \frac{\frac{1}{2}}{1+\sqrt{2}u}\right) du =$$

$$= \frac{1}{2}\int \frac{du}{1-\sqrt{2}u} + \frac{1}{2}\int \frac{du}{1+\sqrt{2}u} =$$

$$\left[\begin{array}{ll} s = 1-\sqrt{2}u & z = 1+\sqrt{2}u \\ ds = -\sqrt{2}\,du & dz = \sqrt{2}\,du \end{array}\right]$$

$$= \frac{1}{2}\int \frac{ds}{-\sqrt{2}\cdot s} + \frac{1}{2}\int \frac{dz}{\sqrt{2}\cdot z} =$$

$$= -\frac{1}{2\sqrt{2}}\ln|s| + \frac{1}{2\sqrt{2}}\ln|z| + C =$$

$$= -\frac{1}{2\sqrt{2}}\ln|1-\sqrt{2}\sin t| + \frac{1}{2\sqrt{2}}\ln|1+\sqrt{2}\sin t| + C =$$

$$= -\frac{1}{2\sqrt{2}}\ln|1-\sqrt{2}\sin \operatorname{arctg} x| + \frac{1}{2\sqrt{2}}\ln|1+\sqrt{2}\sin\operatorname{arctg} x| + C$$

$$= -\frac{1}{2\sqrt{2}}\ln\left|1-\sqrt{2}\frac{x}{\sqrt{1-x^2}}\right| + \frac{1}{2\sqrt{2}}\ln\left|1+\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right| + C$$

$x = \operatorname{tg} t \Rightarrow t = \operatorname{arctg} x$

$$x^2 = \frac{\sin^2 t}{\cos^2 t} = \frac{\sin^2 t}{1-\sin^2 t}$$

$$x^2(1-\sin^2 t) = \sin^2 t$$

$$x^2 - x^2\sin^2 t = \sin^2 t$$

$$x^2 = \sin^2 t\,(1+x)$$

$$\sin t = \frac{x}{\sqrt{1+x}}$$

$\sqrt{1+x^2} \qquad x = tg\, t$

(4.) $$\int \frac{dx}{(x+1)^2\sqrt{x^2+2x+2}} = \int \frac{dx}{(x+1)^2\sqrt{(x+1)^2+1}} = \int \frac{\frac{1}{\cos^2 t}\,dt}{(tg\,t)^2\sqrt{(tg\,t)^2+1}} =$$

$$= \int \frac{\frac{1}{\cancel{\cos^2 x}}\,dt}{\frac{\sin^2 t}{\cancel{\cos^2 t}}\sqrt{\frac{\sin^2 t}{\cos^2 t}+1}} = \int \frac{dt}{\sin^2 t\sqrt{\frac{1}{\cos^2 t}}} = \qquad \begin{matrix} tg\,t = x+1 \\ \frac{1}{\cos^2 t}\,dt = 1\,dx \end{matrix} =$$

$$= \int \frac{dt\,\cos t}{\sin^2 t} = \int \frac{\frac{du}{\cancel{\cos t}}\,\cancel{\cos t}}{u^2} = \int u^{-2}\,du = \frac{u^{-1}}{-1} + C =$$

$$\left[\begin{matrix} u = \sin t \\ du = \cos t\,dt \\ dt = \frac{du}{\cos t} \end{matrix}\right] \qquad = \frac{(\sin t)^{-1}}{-1} + C =$$

$$= -\left(\sin arc\,tg\,(x+1)\right)^{-1} + C$$

(5.) $r = 2\sin\left(2\varphi + \frac{\pi}{6}\right) = 1$

$\sin\left(2\varphi + \frac{\pi}{6}\right) = \frac{1}{2}$

$2\varphi + \frac{\pi}{6} = \frac{\pi}{6} + 2k\pi$

$\qquad = \frac{5\pi}{6} + 2k\pi$

1.) $2\varphi + \frac{\pi}{6} = \frac{\pi}{6} \Rightarrow \varphi = 0$

2.) $2\varphi + \frac{\pi}{6} = \frac{\pi}{6} + 2\pi \Rightarrow \varphi = \pi$

3.) $2\varphi + \frac{\pi}{6} = \frac{5\pi}{6} \Rightarrow \varphi = \frac{2\pi}{6} = \frac{\pi}{3}$

4.) $2\varphi + \frac{\pi}{6} = \frac{5\pi}{6} + 2\pi \Rightarrow \varphi = \frac{4\pi}{6}$

$x = r\cos\varphi = 2\sin\left(2\varphi + \frac{\pi}{6}\right)\cos\varphi$

$y = 2\sin\left(2\varphi + \frac{\pi}{6}\right)\sin\varphi$

$\varphi = 0:$ $\quad x(0) = 2\cos\frac{\pi}{6} \qquad\qquad \varphi = \pi:$

$y(0) = 0$

$x'(0) = ?$

$y'(0) = ?$

$k = \frac{y'(0)}{x'(0)} \qquad y - y(0) = k(x - x(0))$

$$\cos^2 t = \frac{1+\cos 2t}{2}$$

6. $$\int \sqrt{4-x^2}\,dx = \int 2\sqrt{1-\frac{x^2}{4}}\,dx = 2\int \sqrt{1-\left(\frac{x}{2}\right)^2}\,dx =$$

$$= 2\int \sqrt{1-\sin^2 t}\,\cos t\,dt\cdot 2 = 2\int \cos^2 t\,dt \cdot 2 =$$

$$\left[\begin{array}{l} \frac{x}{2} = \sin t \\ \frac{1}{2}dx = \cos t\,dt \end{array}\right]$$

$$= \cancel{4}\int \frac{1+\cos 2t}{\cancel{2}}\,dt =$$

$$= 2\int 1\,dt + 2\int \cos 2t\,dt =$$

$$\left[\begin{array}{l} 2t = u \\ 2\,dt = du \\ dt = \frac{du}{2} \end{array}\right]$$

$$= 2t + 2\int \cos u\,\frac{du}{2} =$$

$$= 2t + \cancel{2}\tfrac{1}{\cancel{2}}\sin u + C =$$

$$= 2t + \sin 2t + C =$$

$$= 2\arcsin\frac{x}{2} + \sin 2\arcsin\frac{x}{2} + C$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{a^2}{2}\arcsin\frac{x}{2} + \frac{x}{2}\sqrt{a^2-x^2} + C$$

7. $$\int \sqrt{e^x+1}\,dx = \int \sqrt{t}\,\frac{dt}{t-1} = \int u\,\frac{2u\,du}{u^2-1} =$$

$$\left[\begin{array}{l} t = e^x+1 \Rightarrow e^x = t-1 \\ dt = e^x\,dx \\ dx = \frac{dt}{e^x} = \frac{dt}{t-1} \end{array}\right] \quad \left[\begin{array}{l} u^2 = t \\ 2u\,du = dt \end{array}\right]$$

$$= \int \frac{2u^2\,du}{u^2-1} = \int 2 + \frac{2}{u^2-1}\,du = \int 2\,du + \int \frac{2}{u^2-1}\,du =$$

$2u^2 : (u^2-1) = 2$

$\underline{2u^2-2}$

2

$$= \int 2\,du + \int \frac{2}{(u-1)(u+1)}\,du =$$

$$= \int 2\,du + \int \frac{1}{u-1}\,du - \int \frac{1}{u+1}\,du$$

$$\frac{2}{(u-1)(u+1)} = \frac{A}{u-1} + \frac{B}{u+1} = \frac{A(u+1)+B(u-1)}{(u-1)(u+1)} =$$

$$= \frac{Au + A + Bu - B}{(u-1)(u+1)}$$

$u:\ A + B = 0 \Rightarrow A = -B$ $\quad \boxed{A = 1}$

$1:\ A - B = 2 \Rightarrow -B - B = 2$

$\boxed{B = -1}$

$(a^x)' = a^x \ln a$

$= 2u + \ln|u-1| - \ln|u+1| + C =$
$= 2\sqrt{e^x+1} + \ln\left|\sqrt{e^x+1} - 1\right| - \ln\left|\sqrt{e^x+1} + 1\right| + C$

(8.) $\int \frac{2^x}{1+4^x}\,dx = \int \frac{t}{1+2^{2x}} \frac{dt}{2^x \ln 2} = \int \frac{\cancel{t}}{1+t^2} \frac{dt}{\cancel{t}\ln 2} =$

$\left[\begin{array}{l} t = 2^x \\ dt = 2^x \ln 2\, dx \\ dx = \frac{dt}{2^x \ln 2} \end{array}\right]$ $\quad 2^{2x} = (2^x)^2$

$= \frac{1}{\ln 2}\int \frac{dt}{1+t^2} = \frac{1}{\ln 2} \cdot \operatorname{arctg} t + C =$

$= \frac{1}{\ln 2} \operatorname{arctg} 2^x + C$

(9) $\int \frac{x\,dx}{\cos^2 x}$

$u = x$ $\qquad dv = \frac{dx}{\cos^2 x}$ $\qquad$ v dv stlacimo ini već strani

$du = \frac{x^2}{2}$ $\qquad v = \operatorname{tg} x$