

1. Gaussova porzdelitev  $N(\mu, \sigma)$  ima srednja vrednost

$\mu = 275,759$  evot in standardno odstopanje  $\sigma = 45,424$  evot, ugotovljenih iz 100 meritev.

$$n = 100$$

$$\sigma = 45,424$$

$$\mu = 275,759$$

a) izračunaj napaki  $\mu$  in  $\sigma$  ter CV

• napaka aritmetične sredine  $\sigma_x$

$$\sigma_x = \bar{x} \pm \frac{s}{\sqrt{n}} \rightarrow \sigma \text{ [evote]}$$

$\mu$

$$\sigma_x = 275,759 \pm \frac{45,424}{\sqrt{100}}$$

$$\sigma_x = 275,759 \pm 4,542$$

$$\rightarrow 271,217 < \mu < 280,301$$

• napaka standardnega odstopanja  $\sigma_s$

$$\sigma_s = s \pm \frac{s}{\sqrt{2n}} \text{ [evote]}$$

$$\sigma_s = 45,424 \pm \frac{45,424}{\sqrt{200}}$$

$$\sigma_s = 45,424 \pm 3,212$$

$$\rightarrow 42,212 < s < 48,636$$

• Variacijski koeficient CV [%]

$$CV = \frac{s}{\bar{x}} \cdot 100 \text{ [%]}$$

$$CV = \frac{45,424}{275,759} \cdot 100 = 16,472 \%$$

b) Kakšna je TOČNOST in koliko venter je potrebno za točnost 1%.

za  $n > 30$ :

$$p = cv \cdot \frac{\lambda}{\sqrt{n}}$$

za  $n < 30$

$$p = cv \cdot \frac{t}{\sqrt{n}}$$

$$p = cv \cdot \frac{\lambda}{\sqrt{n}} \quad [ \% ]$$

$$p = 16,472 \% \cdot \frac{1,960}{\sqrt{100}}$$

$$p = 3,229 \%$$

1,960 sprejeto

$$p \cdot \mu \cdot \sqrt{n} = \lambda \cdot \sigma \cdot 100 \%$$

$$\sqrt{n} = \frac{\lambda \cdot \sigma \cdot 100 \%}{p \cdot \mu}$$

$$\sqrt{n} = \frac{1,960 \cdot 45,424 \cdot 100 \%}{3 \% \cdot 275,759}$$

$$\sqrt{n} = 32,29$$

$$n = 1042$$

Tabela sh 253:

$$S = 95 \% \rightarrow \lambda = 1,960$$

$$S = 99 \% \rightarrow \lambda = 2,576$$

$$S = 99,9 \% \rightarrow \lambda = 3,291$$

KADAR NIMAMO PODANEGA  
STATISTIČNEGA ZAUPANJA,  
VZAMEMO  $S = 95 \%$ .

c) Koefficient Pearson I ( $M = 262,50$ ), kaj pomeni za točnost 1%.

$$M = 262,50$$

$$p = 1 \%$$

$\bar{x}$  najvišja vrednost

$$P_1 = \frac{\mu - M}{\sigma}$$

$s$

+ = asimetrična (porazdelitev z daljšimi "x-pov" proti desno)

- = asimetrična (porazdelitev z daljšimi "x-pov" proti levi)

$$P_1 = \frac{275,759 - 262,50}{45,424} = 0,292 \rightarrow \text{Je asimetrična (porazdelitev z daljšimi "x-pov" proti levi)}$$

d.) kakšna je verjetnost na mejah intervala zaupanja  $\mu$ ?

• INTERVAL ZAUPANJA  $\mu$  za 95%.

$n > 30$   $s$   $n < 30$

$$\mu \pm z \cdot \frac{s}{\sqrt{n}}$$

$$\mu \pm t \cdot \frac{s}{\sqrt{n}}$$

tabela str 253  
(za 95% = 1,960)

$$\mu \pm z \cdot \frac{s}{\sqrt{n}}$$

$$275,759 \pm 1,960 \cdot \frac{45,424}{\sqrt{100}}$$

$$275,759 \pm 8,903$$

$$\rightarrow 266,856 < \mu < 284,662$$

• VERJETNOST

$$z_{1,2} = \frac{\mu - \bar{x}}{s}$$

$$z_1 = \frac{266,856 - 275,759}{45,424}$$

$$z_2 = \frac{284,662 - 275,759}{45,424}$$

$$z_1 = -0,20$$

$$z_2 = 0,20$$

$$\rightarrow \varphi(0,20) = \text{SKRIPTA STR 251: } 0,3910$$

e.) kolikšna so medsebojni enoti pri 10% in 99,5% veli mentor

• PRI 10%



tristo kar ostane, paži, ne 90%!

$$\textcircled{1} \Phi(z) = (0,5 - 0,90) \cdot 2 = -0,8$$

\textcircled{2} TABELA STR 253 (imamo določen  $\Phi(z)$ , kolikšen je  $z$ )

$z$	$\Phi(z)$
1,28	0,7994
	0,8
1,30	0,8064

$$z = - \frac{1,30 - 1,28}{0,8064 - 0,7994} \cdot 0,0064 + 1,30$$

$$z = 1,2817$$

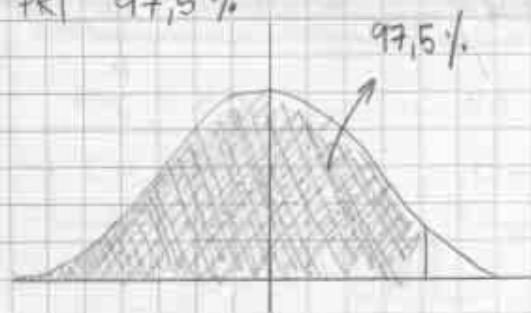
$$\textcircled{3} X_i = \pm z_i \cdot s + \bar{x}$$

↳ paži kaj vsebuje  
in če na dnu na pari

$$X_i = -1,2817 \cdot 45,424 + 275,759$$

$$X_i = 217,539$$

• PRI 97,5%



$$① \Phi(\lambda) = (0,5 - 0,025) \cdot 2 = \underline{0,95}$$

$$② \lambda = 1,960$$

$$③ X_i = \pm \lambda \cdot s + \bar{x}$$

$$X_i = 1,960 \cdot 45,424 + 275,759$$

$$\underline{X_i = 364,790}$$

f.) kakšno je statistično zaupanje intervala zaupanja  $\sigma$ ?

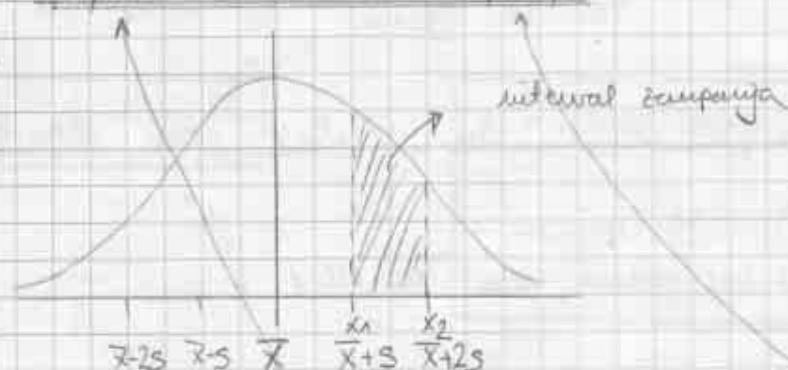
• INTERVAL ZAUPANJA  $\sigma$ :

$$\bar{\sigma} \pm \frac{\sigma}{\sqrt{2n}} \cdot \lambda$$

$$45,424 \pm \frac{45,424}{\sqrt{200}} \cdot 1,960$$

$$45,424 \pm 6,295$$

$$\rightarrow \underline{39,129 < \sigma < 51,719}$$



$$① X_1 = \bar{x} + \sigma_1$$

$$X_1 = 275,759 + 39,129$$

$$\underline{X_1 = 314,888}$$

$$X_2 = \bar{x} + \sigma_2$$

$$X_2 = 275,759 + 51,719$$

$$\underline{X_2 = 327,478}$$

$$② \lambda = \frac{X_i - \bar{x}}{s}$$

$$\lambda_1 = \frac{314,888 - 275,759}{45,424}$$

$$\underline{\lambda_1 = 0,86}$$

$$\lambda_2 = \frac{327,478 - 275,759}{45,424}$$

$$\underline{\lambda_2 = 1,14}$$

③ TABELA STR 253:

$$\Phi(0,86 \div 1,14) = \frac{\Phi(1,14)}{2} - \frac{\Phi(0,86)}{2} = \frac{0,7458}{2} - \frac{0,6102}{2} = \underline{0,0678}$$

④  $S = 100 \cdot \Phi(n)$

$S = 6,78\%$

g.) zgoranja in padaja mase  $\sigma$  za  $S = 99\%$

= INTERVAL ZAUPANJA ZA  $\sigma$  ZA  $S = 99\%$

$$S \pm \frac{S}{\sqrt{n}} \cdot \lambda$$

→ različna glede na štev. skupine

$S = 95\% \rightarrow \lambda = 1,960$

$S = 99\% \rightarrow \lambda = 2,576$

$S = 99,9\% \rightarrow \lambda = 3,291$

$S = 99\% \rightarrow 45,424 \pm \frac{45,424}{\sqrt{12 \cdot 100}} \cdot 2,576$

$\rightarrow 37,150 < \sigma < 53,698$

h.) koliko mentev je znatnej zgoranje in padanje mase torke  $g^2$

= RABINO STATISTIČNO ZAUPANJE INTERVALA  $\sigma$

①  $X_1 = \bar{x} + (\sigma_1)$

$X_2 = \bar{x} + (\sigma_2)$

$X_1 = 275,759 + 37,150$

$X_2 = 275,759 + 53,698$

$X_1 = 312,909$

$X_2 = 329,457$

②  $d = \frac{x_i - \bar{x}}{s}$

$d_1 = \frac{312,909 - 275,759}{45,424}$

$d_2 = \frac{329,457 - 275,759}{45,424}$

$d_1 = 0,82$

$d_2 = 1,18$

③ TABELA STR 253:

$$\Phi(0,82 \div 1,18) = \frac{\Phi(1,18)}{2} - \frac{\Phi(0,82)}{2} = \frac{0,7620}{2} - \frac{0,5878}{2} = \underline{0,0871}$$

④  $S = 100 \cdot \Phi(n)$

$S = 8,71\%$

⑤  $\frac{100 \dots 100\%}{x \dots 8,71\%}$

$N = 9$



i) zgornja in spodnja meja  $\sigma$  za  $S = 95\%$  in le 29 meritev

$$S \pm \frac{S}{\sqrt{2n}} \cdot \lambda \quad \rightarrow \text{razbina glede na stat. zaupanje}$$

$$S = 95\% \rightarrow \lambda = 1,960$$

$$S = 99\% \rightarrow \lambda = 2,576$$

$$S = 99,9\% \rightarrow \lambda = 3,291$$

$$S = 95\% \rightarrow 45,424 \pm \frac{45,424}{\sqrt{2 \cdot 29}} \cdot 1,960$$

$$45,424 \pm 11,690$$

$$\rightarrow 33,734 < \sigma < 57,114$$

j) Koliko meritev  $x_i$  znotraj zgornje in spodnje meje  $\sigma$   
2 točke 12

= RABINO STATIST. ZAUPANJE INTERVALA  $\sigma$

$$\textcircled{1} X_1 = \bar{x} + \sigma_1$$

$$X_2 = \bar{x} + \sigma_2$$

$$X_1 = 275,759 + 33,734$$

$$X_1 = 309,493$$

$$X_2 = 275,759 + 57,114$$

$$X_2 = 332,873$$

$$\textcircled{2} \lambda_{1,2} = \frac{x_i - \bar{x}}{S}$$

$$\lambda_1 = \frac{309,493 - 275,759}{45,424}$$

$$\lambda_2 = \frac{332,873 - 275,759}{45,424}$$

$$\lambda_1 = 0,74$$

$$\lambda_2 = 1,26$$

$\textcircled{3}$  TABELA SR: 253

$$\Phi(0,74 \div 1,26) = \frac{\Phi(1,26)}{2} - \frac{\Phi(0,74)}{2} = \frac{0,7924}{2} - \frac{0,5408}{2} = 0,1258$$

$$\textcircled{4} S = \Phi(\lambda) \cdot 100$$

$$S = 12,58\%$$

$$\textcircled{5} \begin{array}{ll} 29 \dots 100\% \\ N \dots 12,58\% \end{array}$$

$$N = 4$$

b.) kakšen je % mentor med 250 in 300 enotama.

①  $X_1 = 250$   
 $X_2 = 300$

pretvorba v standardno

$$\hat{x}_1 = \frac{x_1 - \bar{x}}{s} = \frac{250 - 275,759}{45,424} = -0,57$$

$$\hat{x}_2 = \frac{300 - 275,759}{45,424} = 0,53$$

②  $\Phi(-0,57 \div 0,53) = \frac{\Phi(0,53)}{2} - \left( -\frac{\Phi(0,57)}{2} \right) = \frac{0,4039}{2} + \frac{0,4313}{2} = 0,4176$

$\hat{x}$	$\Phi(\hat{x})$
0,52	0,3970
0,53	
0,54	0,4108

$$\Phi(\hat{x}) = -\frac{0,4108 - 0,3970}{0,54 - 0,52} \cdot 0,01 + 0,4108 = 0,4039$$

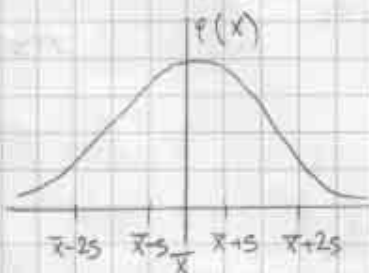
$\hat{x}$	$\Phi(\hat{x})$
0,56	0,4246
0,57	
0,58	0,4380

$$\Phi(\hat{x}) = -\frac{0,4380 - 0,4246}{0,58 - 0,56} \cdot 0,01 + 0,4380 = 0,4313$$

③  $S = \Phi(\hat{x}) \cdot 100$   
 $S = 41,76\%$

l.) Kakšna je pogostost porazdelitve  $\mu$ ,  $\mu \pm 0,5\sigma$ ,  $\mu \pm \sigma$ ,

$\mu \pm 2\sigma$ ,  $\mu \pm 3\sigma$  ?



POGOSTOST  
 PORAZDELITVE

$$f(x) = \varphi(x) \cdot H$$

predstavlja največjo  
 vrednost pri dani vrednosti

$$\varphi(x) = \left[ \frac{1}{\sigma \cdot \sqrt{2\pi}} \right] = \gamma_{\max}$$

$\mu \rightarrow \varphi(x) = \frac{1}{45,424 \cdot \sqrt{2\pi}} = 0,0088 \rightarrow f(x) = 0,0088 \cdot 100 = 0,88$

TABELA STR 252:

$\mu \pm 0,5\sigma \rightarrow \varphi(x) = \frac{7}{8} \gamma_{\max}$

$$\varphi(x) = \frac{7}{8} \cdot \frac{1}{\sigma \cdot \sqrt{2\pi}} = \frac{7}{8} \cdot \frac{1}{45,424 \cdot \sqrt{2\pi}} = 0,0077$$

$$\rightarrow f(x) = 0,0077 \cdot 100 = 0,77$$

•  $\mu \pm \sigma$

$$\varphi(x) = \frac{5}{8} \cdot \gamma_{\max}$$

$$\begin{aligned} \varphi(x) &= \frac{5}{8} \cdot \frac{1}{\sigma \cdot \sqrt{2\pi}} = \\ &= \frac{5}{8} \cdot \frac{1}{45,424 \cdot \sqrt{2\pi}} = \\ &= 0,0055 \end{aligned}$$

$$\begin{aligned} \rightarrow \varphi(x) &= \varphi(x) \cdot 100 \\ &= 0,0055 \cdot 100 \\ &= \underline{\underline{0,55}} \end{aligned}$$

•  $\mu \pm 2\sigma$

$$\varphi(x) = \frac{1}{8} \cdot \gamma_{\max}$$

$$\begin{aligned} \varphi(x) &= \frac{1}{8} \cdot \frac{1}{\sigma \cdot \sqrt{2\pi}} = \\ &= \frac{1}{8} \cdot \frac{1}{45,424 \cdot \sqrt{2\pi}} = \\ &= 0,0011 \end{aligned}$$

$$\begin{aligned} \rightarrow \varphi(x) &= \varphi(x) \cdot 100 \\ &= 0,0011 \cdot 100 \\ &= \underline{\underline{0,11}} \end{aligned}$$

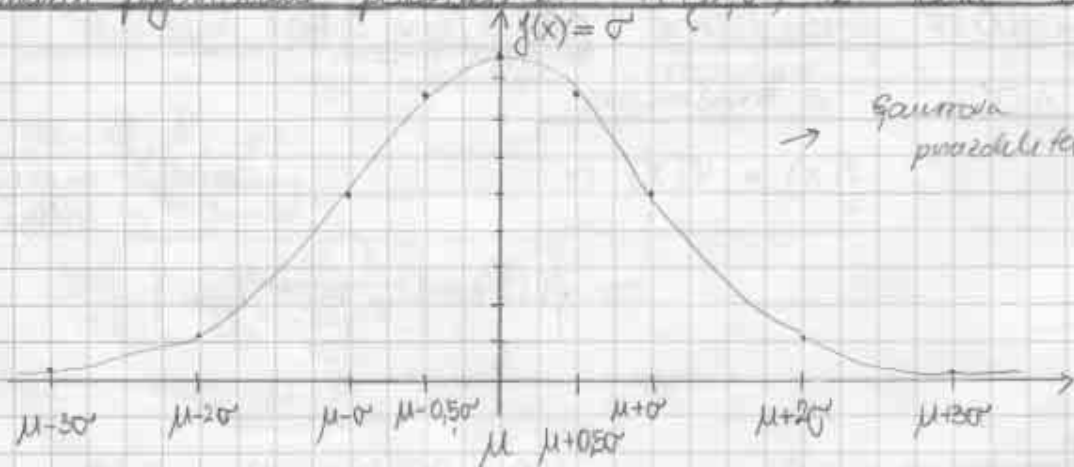
•  $\mu \pm 3\sigma$

$$\varphi(x) = \frac{1}{80} \cdot \gamma_{\max}$$

$$\begin{aligned} &= \frac{1}{80} \cdot \frac{1}{45,424 \cdot \sqrt{2\pi}} = \\ &= 0,0001 \end{aligned}$$

$$\begin{aligned} \rightarrow \varphi(x) &= \varphi(x) \cdot 100 \\ &= 0,0001 \cdot 100 \\ &= \underline{\underline{0,01}} \end{aligned}$$

m.) ~~normale prozentrations prozentrations~~  $N(\mu, \sigma)$  iz tike 1.





n) ali so sprejemljive razlike med  $\mu$  in dogovorjeno vrednost 10<sup>3</sup> 300 evrov?

→ KATERA HIPOTEZA VELJA

$$\mu = 275,759$$

$$\mu = 300 \text{ (dogovorjena vrednost)}$$

DOGODVORJENA  
VREDNOST  $\mu$   
OZ  
PREDPISANA VREDNOST

$$t = \frac{|\bar{x} - \mu|}{s} \cdot \sqrt{n}$$

$$t = \frac{|275,759 - 300|}{45,424} \cdot \sqrt{100}$$

$$t = 5,34$$

$$t > t_t$$

$$a) t \leq t_t (S=95\%, v) \rightarrow H_0 (\bar{x}_1 = \bar{x}_2)$$

$$b) t \geq t_t (S=99\%, v) \rightarrow H_1 (\bar{x}_1 \neq \bar{x}_2)$$

$$c) t_t (S=99\%, v) > t > t_t (S=95\%, v)$$

$$v = n - 1$$

$$v = 99$$

④ TABELA STR 254

$$a) 5,34 \leq 1,984 \quad //$$

$$b) 5,34 \geq 2,626 \quad \checkmark$$

$$c) 2,626 > 5,34 > 1,984$$

v	t
90	1,987
99	
100	1,984

$$t = - \frac{1,984 - 1,987}{100 - 90} \cdot 1 + 1,984$$

$$= 1,9843$$

v	t
90	2,631
99	
100	2,626

$$t = - \frac{2,626 - 2,631}{10} \cdot 1 + 2,626$$

$$t = 2,626$$

⑤ Velja alternativna hipoteza, zato razlike niso sprejemljive!

0.) ali so sprejemljive razlike med  $\sigma$  in dogovorjeno vrednostjo 30 evrov?

① 
$$Z = \frac{\sigma}{s}$$

predpisano standardno odstopanje  $\sigma$   
 sipanje  $s$

DOGOVORJENA  
VREDNOST  $\sigma$

$$Z = \frac{30}{45,424} = 0,66$$

②  $Z < 1, \sigma < s \rightarrow$  GLEJ KRIVULJE  $Z_u$   
 $Z > 1, \sigma > s \rightarrow$  GLEJ KRIVULJE  $Z_o$

DIAGRAM STR 267

PAZI! LEVA IN DESNA  
ORDINATA !!!

③ gledam krivuljo  $Z_u$

a)  $Z \leq Z_u (s = 95\%) \rightarrow H_0: s = \sigma$

b)  $Z \geq Z_u (s = 99\%) \rightarrow H_1: s \neq \sigma$

c)  $Z_u (s = 99\%) > Z > Z_u (s = 95\%)$

④ a)  $0,66 \leq 0,89 \checkmark$

b)  $0,66 \geq 0,85 //$

c)  $0,85 > 0,66 > 0,89$   
 $\checkmark \quad \quad \quad \downarrow$

⑤ velja ničelna hipoteza, zato so razlike sprejemljive

p) ali  $\mu$  in  $\sigma$  ustrezata vrednostima 300 evrov in 30 evrov?

$\mu = 300$  evrov ne ustreza, ker velja  $H_1$

$\sigma = 30$  evrov ustreza, ker velja  $H_0$ .

• NIČELNA

HIPOTEZA  $H_0$ :

$$(\bar{X}_1 = \bar{X}_2)$$

nežlike med  
in tvorčino  
(razlike so)

poravnano, ki  
povzročajo  
sprejemljive

ga  
ga  
skladno  
načrtovanje  
 $(\bar{X}_1 \neq \bar{X}_2)$

• ALTERNATIVNA

razlike med  
in tvorčino  
(razlike niso)

poravnano, ki  
povzročajo  
sprejemljive

ga  
ga  
skladno  
načrtovanje  
dokazne